Thermomechanical analysis of functionally graded laminates using tolerance approach

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Abstract

Some thermoelasticity problems in laminates made of two components non-periodically distributed as microlaminas along one direction are presented in this note. To obtain the governing equations describing these problems, the tolerance averaging technique is applied. In this paper, three models are proposed: the tolerance and the asymptotic-tolerance model, taking into account the effect of the microstructure size on the overall behaviour of these structures, and the asymptotic model, neglecting this effect. To solve the equations of these models the Finite Difference Method is used.

Keywords: thermomechanics, functionally graded laminates, tolerance approach, microstructure, Finite Difference Method

1. Introduction

In this paper we deal with the problems of thermoelasticity in laminates, made of two materials, non-periodically distributed as microlaminas along one direction. The cells are composed of two sublayers of different components. It also assumed that the macroscopic properties of these structures are changing continuously along one direction (perpendicular to the laminas). The thickness of the cells is constant and denoted by λ, as shown in the Fig. 1. These kind of laminates can be called the functionally graded laminates [2].

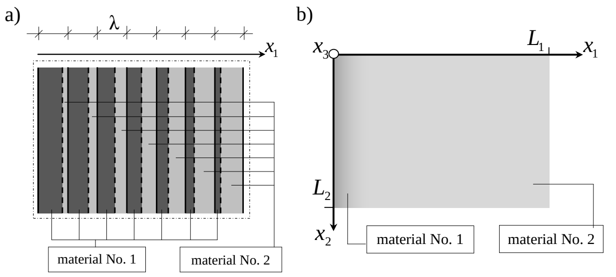

Figure 1: The cross-section of considered laminates: a) microstructure, b) macrostructure

Thermomechanical problems of non-periodic laminates can be described by using some averaging methods, which are used for periodic composites [8]. Unfortunately, the effect of the microstructure size on the overall behaviour of the laminates is usually neglected in the proposed model equations.

In order to obtain the equations, that take into account this effect, the tolerance averaging technique is used [9]. This method was proposed and used to consider various thermomechanical problems of periodic media [1,9]. The tolerance approach was also adopted to model problems of composite structures, made of materials with functional gradation of properties. This technique was applied to analyse e.g. heat conduction problems [4,5,6], thermoelasticity problems [2,7], vibrations of laminates with thermal effects [3].

By using the tolerance modelling we can replace the system of differential equations with functional, highly oscillating, tolerance-periodic and non-continuous coefficients, by equations where the coefficients are slowly-varying or constant. This technique is based on the concepts of a slowly-varying function, a tolerance-periodic function and an averaging operation.

The equations of three models are obtained. Two of them – the tolerance and the asymptotic-tolerance model, describe the effect of the microstructure size on the overall behaviour of the functionally graded laminates, and the third – asymptotic model, neglects this effect.

2. The modelling procedures

Thermoelasticity problems of composites can be described by Eqns (1):

$$\partial_j (C_{ijkl}\partial_l u_k - \rho\ddot{u}_i = \partial b_{ij}\theta + b_{ij}\partial_j\theta$$

$$\partial_j (k_{ij}\partial_i\theta) = c\dot{\theta} + T_0 b_{ij}\partial_j \dot{u}_i \qquad (1)$$

where C_{ijkl}, ρ, b_{ij}, k_{ij}, c are highly-oscillating, non-continuous functional coefficients of x_1 for displacements u_i (i,j,k,l=1,2,3) and a temperature θ.

The tolerance approach is based on two fundamental assumptions. The first is the micro-macro decomposition assumed in the form Eqns (2):

$$u_i(x,\mathbf{x},t) = w_i(x,\mathbf{x},t) + h(x)v_i(x,\mathbf{x},t)$$

$$\theta(x,\mathbf{x},t) = \vartheta(x,\mathbf{x},t) + g(x)\psi(x,\mathbf{x},t) \qquad (2)$$

where: $x \equiv x_1$, $\mathbf{x} \equiv (x_1,x_2)$ and w_i, v_i, ϑ, ψ are slowly-varying functions in x. Functions w_i and θ are the basic unknowns, called the macrodisplacements and the macrotemperature, respectively, v_i and ψ are additional basic unknowns, called the fluctuation amplitudes of displacements and of temperature, respectively, $h(x)$ and $g(x)$ are the fluctuation shape functions, assumed as saw-like functions.

The second assumption is the tolerance averaging approximation in which terms of an order of $O(\delta)$ are assumed to be negligibly small.

Substituting micro-macro decomposition to governing equations of thermoelasticity, doing some averaging and appropriate manipulations, the final equations of the tolerance model can be derived.

The asymptotic model equations can be obtained directly from the equations of the tolerance model, by neglecting the terms, which depend on the microstructure size.

The asymptotic-tolerance modelling procedure can be carried out in two steps. In the first step, the asymptotic solution is obtained in the form Eqns (3):

$$u_{0i}(x,\mathbf{x},t)=w_i(x,\mathbf{x},t)+h(x)v_i(x,\mathbf{x},t)$$

$$\theta_0(x,\mathbf{x},t)=\vartheta(x,\mathbf{x},t)+g(x)\psi(x,\mathbf{x},t) \qquad (3)$$

and the second step is using the additional micro-macro decomposition, Eqns (4), to Eqns (1):

$$u_i(x,\mathbf{x},t)=w_i(x,\mathbf{x},t)+f(x)r_i(x,\mathbf{x},t)$$

$$\theta(x,\mathbf{x},t)=\vartheta(x,\mathbf{x},t)+d(x)\chi(x,\mathbf{x},t) \qquad (4)$$

with w_i, v_i, ϑ, ψ as known functions, r_i, χ as new unknowns being slowly-varying functions and f, d as new additional fluctuation shape functions similar to h, g.

3. The model equations

Using various modelling procedures, governing equations of three averaged models can be obtained. The underlined terms in equations of the tolerance model, Eqns (5), depend on the microstructure size:

$$\partial_j(<c_{ijkl}>\partial_l w_k+<c_{ijk1}\partial h>v_k)-<\rho>\ddot{w}_i=\partial<b_{i1}>\vartheta+<b_{ij}>\partial_j\vartheta$$
$$-\underline{c_{i\alpha k\beta}hh>\partial_\alpha\partial_\beta v_k}+<c_{i1k1}\partial h\partial h>v_k+<c_{i1kl}\partial h>\partial_l w_k+$$
$$+\underline{<\rho hh>\ddot{v}_i}=-<b_{i1}\partial g>\vartheta+\underline{<b_{i\beta}gh>\partial_\beta\psi}$$
$$\partial_j(<k_{ij}>\partial_i\vartheta+<k_{1j}\partial g>\psi)=<c>\dot{\vartheta}+<T_0 b_{ij}>\partial_j\dot{w}_i+<T_0 b_{i1}\partial h>\dot{v}_i$$
$$\underline{<k_{\alpha\beta}gg>\partial_\alpha\partial_\beta\psi}-<k_{i1}\partial g>\partial_i\vartheta-<k_{11}\partial g\partial g>\psi=$$
$$=\underline{<cgg>\dot{\psi}}+\underline{<T_0 b_{i\beta}hg>\partial_\beta\dot{v}_i} \qquad (5)$$

where all coefficients in <•> are functional but slowly-varying and smooth.

By limit passage with λ to zero, the equations of the asymptotic model are obtained, Eqns (6):

$$\partial_j(<c_{ijkl}>\partial_l w_k+<c_{ijk1}\partial h>v_k)-<\rho>\ddot{w}_i=\partial<b_{i1}>\vartheta+<b_{ij}>\partial_j\vartheta$$
$$<c_{i1k1}\partial h\partial h>v_k+<c_{i1kl}\partial h>\partial_l w_k=-<b_{i1}\partial g>\vartheta$$
$$\partial_j(<k_{ij}>\partial_i\vartheta+<k_{1j}\partial g>\psi)=<c>\dot{\vartheta}+<T_0 b_{ij}>\partial_j\dot{w}_i+<T_0 b_{i1}\partial h>\dot{v}_i$$
$$-<k_{i1}\partial g>\partial_i\vartheta-<k_{11}\partial g\partial g>\psi=0 \qquad (6)$$

which can be obtained directly from equations of the tolerance model, neglecting underlined terms (involving the microstructure parameter λ).

The equations of the asymptotic-tolerance model, Eqns (7), are consisting of the equations of the asymptotic model and the additional equations:

$$\partial_j(<c_{ijkl}>\partial_l w_k+<c_{ijk1}\partial h>v_k)-<\rho>\ddot{w}_i=\partial<b_{i1}>\vartheta+<b_{ij}>\partial_j\vartheta$$
$$\partial_j(<k_{ij}>\partial_i\vartheta+<k_{1j}\partial g>\psi)=<c>\dot{\vartheta}+<T_0 b_{ij}>\partial_j\dot{w}_i+<T_0 b_{i1}\partial h>\dot{v}_i$$
$$<c_{i1k1}\partial h\partial h>v_k=-<c_{i1kl}\partial h>\partial_l w_k-<b_{i1}\partial g>\vartheta$$
$$<k_{11}\partial g\partial g>\psi=-<k_{i1}\partial g>\partial_i\vartheta$$
$$\underline{<c_{i\alpha k\beta}ff>\partial_\alpha\partial_\beta r_k}-\underline{<\rho ff>\ddot{r}_i}-<c_{i1k1}\partial f\partial f>r_k=<c_{i1k1}\partial h\partial f>v_k+$$
$$+<c_{i1kl}\partial f>\partial_l w_k+<b_{i1}\partial f>\vartheta$$
$$\underline{<k_{\alpha\beta}dd>\partial_\alpha\partial_\beta\chi}-<k_{11}\partial d\partial d>\chi=<k_{i1}\partial d>\partial_i\vartheta+<k_{11}\partial g\partial d>\psi+$$
$$+\underline{<cdd>\dot{\chi}}+\underline{<T_0 b_{i\beta}fd>\partial_\beta\dot{r}_i}. \qquad (7)$$

4. Remarks

General and specific remarks can be formulated:

- by using the tolerance approach it is possible to replace the differential equations of thermoelasticity with tolerance-periodic, highly-oscillating, non-continuous coefficients by the differential equations with smooth, slowly-varying coefficients,
- the tolerance and the asymptotic-tolerance model equations take into account the effect of the microstructure size,
- the asymptotic model neglects this effect,
- the equations of these three models can be applied in the analysis of some specific cases, namely, where distribution of the ingredients is functional, but non-periodic.

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