The influence of the response functions on the diagrid and orthogonal grillages reliability by the Stochastic iterative perturbation-based Finite Element Method

Bartłomiej M. Pokusiński[1] **and Marcin M. Kamiński**[2]

[1] *Department of Structural Mechanics, Łódź University of Technology*
Al. Politechniki 6, 90-924 Łódź, Poland
e-mail: bartlomiej.pokusinski@gmail.com

[2] *Department of Structural Mechanics, Łódź University of Technology*
Al. Politechniki 6, 90-924 Łódź, Poland
e-mail: marcin.kaminski@p.lodz.pl

Abstract

The main aim of this elaboration is to verify the influence of the response functions in the stochastic perturbation-based iterative approach on the probabilistic moments and coefficients of the grillage structures. The second purpose is to compare their two different types, orthogonal and diagrid, in terms of the time-independent reliability including an uncertainty in steel Young modulus. We have considered different performance functions concerning the basic eigenfrequency, the global extreme vertical deflection and also local deformation.

Keywords: reliability index, iterative stochastic perturbation technique, response function method, steel diagrid structures

1. Introduction

The diagrid structural systems have come out as one of the most efficient, most adaptable and most innovative approaches to the structuring buildings of this century [1,4]. Due to an aesthetic potential and to the increasing architectural popularity it seems therefore essential to examine its reliability and, additionally compare it with this obtained for the traditional, orthogonal structure. Computer analysis of civil engineering structures with random parameters has remarkably increasing influence on structural design process, optimization and reliability modelling because of variety of uncertainty sources. In the case of the grillages examined Young modulus, as one of the most important parameter, uncertainty have been considered. A solution of the structural problem including randomness is finished with the reliability indices verification, according to the general rules included in Eurocode [2]. Their determination has been provided here by an application of the generalized iterative stochastic perturbation technique [3] and the response function method. Computational implementation of this method is carried out using the FEM civil engineering system ROBOT, where the coefficients of dozen different sets of responses are computed in the computer algebra system MAPLE from several solutions of the original problem obtained for Young modulus varying about its expectation (210 GPa). This form leads to determination of all their partial derivatives with respect to stochastic input variable which can be further used for determination of probabilistic moments and final values of the reliability indices. Finally, the results may be compared to find out (a) which polynomial-based response provides the best correctness and (b) which structure is more efficient in terms of the given uncertainty.

2. The iterative stochastic perturbation technique

The basic idea of the stochastic perturbation approach is an expansion of all random functions of the given problem via Taylor series of the required order about their expectations using perturbation parameter ε. In the case of some real function $f(b)$ of the stationary input random variable $b(\omega)$ with a symmetric probability density function, one can show that in the 10th order perturbation approach the expectation of the structural response function of course remains the same in both linearized and in the nonlinear iterative schemes:

$$E\left[f(b),b\right] \approx f\left(b^0\right) + \sum_{m=1}^{5} \frac{1}{2m!}\varepsilon^{2m} \left.\frac{\partial^{2m} f(b)}{\partial b^{2m}}\right|_{b=b^0} \mu_{2m}(b^0) \tag{1}$$

where $\mu_{2m}(b)$ denotes $2m$th order central probabilistic moment of the quantity b.

Next three probabilistic moments are obtained by including Taylor expansion of the tenth order valid for the expected value into their well-known integral definitions. As a result we obtain some extra components in comparison with the linearized version of stochastic perturbation technique, which can be represented for the variance as follows:

$$\begin{aligned} Var_{iter}\left[f(b)\right] - Var_{linear}\left[f(b)\right] \approx & -\mu_2(b^0)\cdot\left\{\frac{1}{4}\cdot\left[\frac{\partial^2 f(b)}{\partial b^2}\right]^2\cdot\mu_2(b^0) + \right.\\ & + \frac{1}{24}\cdot\frac{\partial^4 f(b)}{\partial b^4}\cdot\frac{\partial^2 f(b)}{\partial b^2}\cdot\mu_4(b^0) + \\ & + \frac{1}{720}\cdot\frac{\partial^6 f(b)}{\partial b^6}\cdot\frac{\partial^2 f(b)}{\partial b^2}\cdot\mu_6(b^0) + \\ & \left. + \frac{1}{40320}\cdot\frac{\partial^8 f(b)}{\partial b^8}\cdot\frac{\partial^2 f(b)}{\partial b^2}\cdot\mu_8(b^0)\right\} + \\ & - \mu_4(b^0)\cdot\left\{\frac{1}{576}\cdot\left[\frac{\partial^4 f(b)}{\partial b^4}\right]^2\cdot\mu_4(b^0) + \right.\\ & \left. + \frac{1}{8640}\cdot\frac{\partial^6 f(b)}{\partial b^6}\cdot\frac{\partial^4 f(b)}{\partial b^4}\cdot\mu_6(b^0)\right\} \end{aligned} \tag{2}$$

3. Response Function Method

As shown above, one of the crucial issues during applying the iterative perturbation based approach is numerical determination of partial derivatives of the structural response function of up to *n*th order with respect to the randomized parameter. To complete this task, it is necessary to determine such a function by a multiple solution of the boundary value problem around the expectation of the random parameter, in the interval $\left[b^0 - \Delta b, b^0 + \Delta b\right]$. Each unknown response function is approximated here by the dozen different sets of responses computed using the Least Squares Method as well as its weighted version (WLSM). We have used: B-spline curve, analytical dependence and polynomials of several orders: 10th, maximizing correlation, minimizing variance, maximizing their quotient and also maximizing correlation between the LSM and the WLSM solution.

4. Finite Element Analysis

All the numerical tests have been performed on the two types, but four examples of the grillages: orthogonal (Model O) and three diagrids (Model Dm - medium, Model Db - the biggest, Model Ds - the smallest grid), presented correspondingly in Fig. 1 a-d.

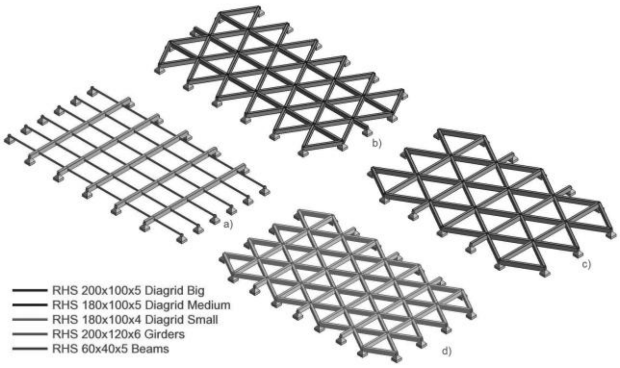

Figure 1: Examples of the grillage structure: a) orthogonal, b) diagrid medium, c) diagrid big, d) diagrid small

The 3D beam finite elements and rigid connection have been used in the mesh together with simple supports with all the linear displacements fixed.

5. Computational reliability analysis

The analyzed response functions have meaningful influence on the probabilistic estimators. The differences are considerable and only for the input coefficient of variation of Young modulus (α) less than 0.025 results are similar, excluding skewness and kurtosis for B-Spline curves.

The criterion of minimizing variance have provided correctness in the narrowest interval. Better results have been obtained for maximizing quotient of correlation and variance, but the best for maximizing correlation (both for the WLSM and the LSM).

Excluding the least accurate criterion, the weighted least-squares method have provided much more relevant results. Polynomials of optimal degree have additionally proved to be better than for 10th order contradicting assumption that higher order response will more precisely reflect the structure behaviour.

Analytical dependencies have turn out to be hardly sensitive to weights. Therefore, the criterion of maximizing correlation between the LSM and the WLSM solution have been created. Such polynomials have proved to be relevant in the widest interval of α (Fig. 2).

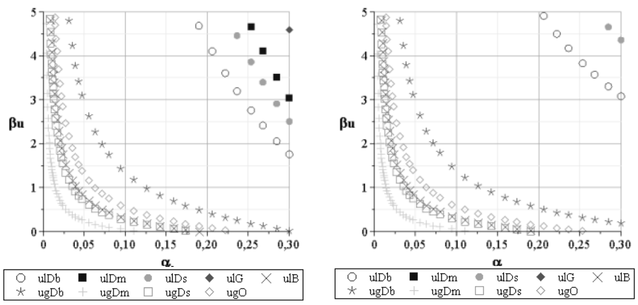

Figure 2: Comparison of the results on the example of reliability index of maximum of the vertical deflection (left-analytical dependences, right-optimal polynomial)

Computational analysis provided in this paper shows that the grillage from Model Db is the most reliable in case of the examined structures. Admittedly, the SLS ratio is the lowest, the weight of structure is lower than for the rest of diagrids anyway. Although Model Db is almost 79% heavier than Model O, the decisive reliability index is at least 3.5 times bigger (Fig. 3), which makes such a structure approximately twice much effective in terms of reliability with Young modulus as input random variable in case of the examined structures.

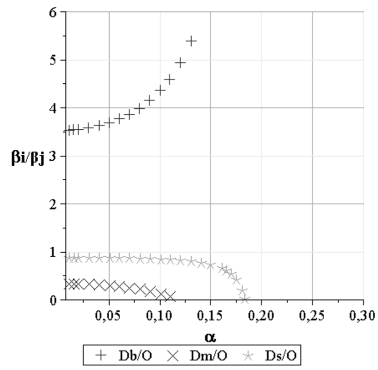

Figure 3: Ratio of the minimal reliability indices

References

[1] Boake, T. M., Diagrid Structures: Innovation and Detailing, *Structures and Architecture: New concepts, applications and challenges*, Cruz P. J. S. Ed., CRC Press, London, 2262, pp. 991-998, 2013.

[2] The European Union Per Regulation, *Eurocode: Basis of structural design. EN 1990*, 2002.

[3] Kamiński, M. M., *The Stochastic Perturbation Method for Computational Mechanics*, Wiley, Chichester, 2013.

[4] Moon, K., Design and Construction of Steel Diagrid Structures, *Proceedings - Nordic Steel Construction Conference 2009*, Lagerqvist O. Ed., Swedish Institute of Steel Construction SBI, Stockholm, 583, pp. 398-405, 2009.