Mechanical characterization of the calcium deposit based on the micro-indentation test and the finite element method computations

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Abstract

In the literature there is a lack of deepened studies where artery calcifications are tested for delivering more reliable material behaviour. This paper combines a micro-indentation test output with the finite element method computations in order to extract more data from a single examination. The inverse problem is solved for linear elastic-plastic model (with nonlinear isotropic hardening; Huber-Mises-Hencky criteria) of the calcium deposits sample. The solution is obtained by introducing an optimization technique in order to decrease the discrepancy between the experimental measurable quantities and its numerical counterparts. The output of ~~the~~ characterized model parameters is presented with the reference to the experimental values.

Keywords: micro-indentation, calcium deposits, inverse problem, finite element method, biomechanics

1. Introduction

The calcium deposits are forming in an advanced stadium of atherosclerosis in histopathologically changed walls of the arteries. In vivo mechanical tests of calcifications are impossible to conduct, therefore the presented technique is an ex vivo method.

Many researchers for representing the mechanical behaviour of calcium deposits use the simplest material models due to the lack of experimental data. Usually linear elasticity or hyperelasticity is used, like in the paper of Wang et al. [3].

This paper describes the acquisition of the material parameter setup for the linear elastic model with nonlinear plastic hardening with Huber-Mises-Hencky criterion. The parameters were identified based on the mixed approach of the micro-indentation test enriched with the finite element method (FEM) computations. The characterization is performed by finding the solution of the inverse problem, similar to the one proposed in the paper of Gajewski and Garbowski [1].

2. Method and model

The general idea of solved inverse problem is presented in Fig. 1. Here, the classical approach is used while the difference between experimental measurable quantities and numerical counterparts is minimized. To obtain the representative mechanical parameters from the inverse analysis several steps are required: the definition of the constitutive model (1), the acquisition of the experimental measurable quantities (2), building FEM model (3), obtaining its numerical counterparts from FEM model (4), the definition of the minimization criterion (5) engaged in an optimization procedure (6).

The material model (1) which is used for characterization is expressed by the formula:

$$\sigma = \begin{cases} E \cdot \varepsilon & \varepsilon < \dfrac{\sigma_0}{E} \\ \sigma_0 + \dfrac{1000}{A^2}\left(\varepsilon - \dfrac{\sigma_0}{E}\right)^{1/B} & \varepsilon \geq \dfrac{\sigma_0}{E} \end{cases} \qquad (1)$$

The unknown material parameters x^i and therefore selected to identify are: E – the Young modulus, σ_0 – the yield stress and the nonlinear plastic hardening parameters A and B.

The experimental procedure (2) was adapted according to the paper of Kot et al. [2], however the Vickers protocol was applied. The maximum force was set to *0.05* N. The indentation curve Q_j^{EXP}, reaction force vs. penetration, was obtained from the experiment. Moreover, based on experimental images two shape parameters of permanent deformation, I_j^{EXP} were determined, i.e. a – the half of imprint shape side and b – the half of shape diameter.

The micro-indentation test was later simulated in inverse analysis loop by three-dimensional FEM model (3). The calcium deposit sample was represented by 8-node brick elements with linear shape functions. The indenter tip was modelled by the 8-node brick elements with linear shape functions. The diamond indenter was modelled with linear elastic constitutive law (E = 1141 GPa, ν = 0.07).

In the inverse problem computations the numerical counterparts of experimental outputs were calculated, i.e. Q_j^{NUM} and I_j^{NUM} for specified sets of the searched parameters, x^i.

The multi-object optimization problem, fitting the curves and the imprint shape, is reduced to the scalar objective minimization by Eq. (2) (the weight w is introduced in order to scale the part of imprint shape in relation to the indentation curve contribution). The cost function in least square form used in the study read the following formulae:

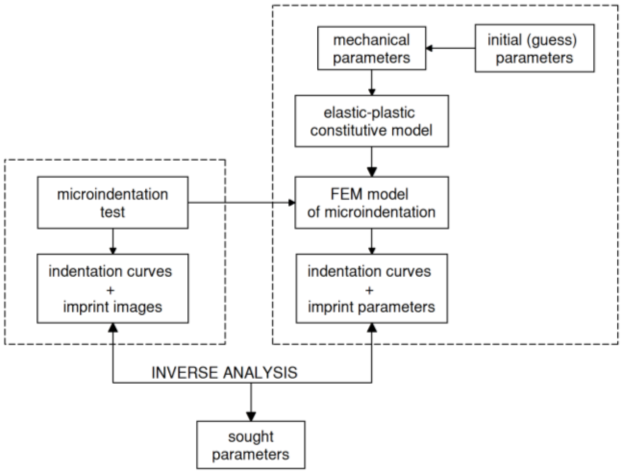

Figure 1: The inverse problem framework of the calcium deposit mechanical characterization

$$f(x^i) = \sum_{i=1}^{n} (Q_j^{EXP} - Q_j^{NUM}(x^i))^2 + \\ + w \cdot \sum_{i=1}^{n} (I_k^{EXP} - I_k^{NUM}(x^i))^2 \quad (2)$$

The constrained trust region algorithm was used as the optimization method (6). Based on the initial parameter guess (see Fig. 1), the algorithm decreases the discrepancy between the experimental and numerical field by updating the mechanical parameters x^i. The solution of the selected optimization method may be the optimal in the local parameter space, therefore multi-start approach was used for determining the optimum in a global sense.

3. Results

The inverse problem was submitted for seven initial point guesses, x_0^i. The derived material parameters x_{opt}^i of four cases were qualified as the successful results. In those four cases the cost function, $f(x_{opt}^i)$ obtained average value of 2.33 with standard deviation of 0.02. The average optimal parameter set x_{opt}^a [E^a, σ_0^a, A^a, B^a] equals [*14160* MPa, *265.2* MPa, *2.286*, *1.360*], respectively.

The numerical output for the final/average solution x_{opt}^a is presented in Fig. 2B,C compared with the experimental quantities. Figure 2A shows the constitutive relation obtained by x_{opt}^a (the final/average parameter set).

4. Conclusions

The study presents the successful mechanical identification of the calcium deposit sample for the linear elastic-plastic constitutive law (with nonlinear hardening, Huber-Mises-Hencky plastic criterion). The experimental data from micro-indentation test are reflected by the FEM computations, see Fig. 2B-C.

Moreover, the indentation modulus E^{IT}, (interpreted as the elastic modulus), given by the micro-indenter device equals *14400* MPa. This is in good correlation with identified (in the study) value of Young modulus E^a (the difference is less than 1.7%). This fact makes the study results more reliable.

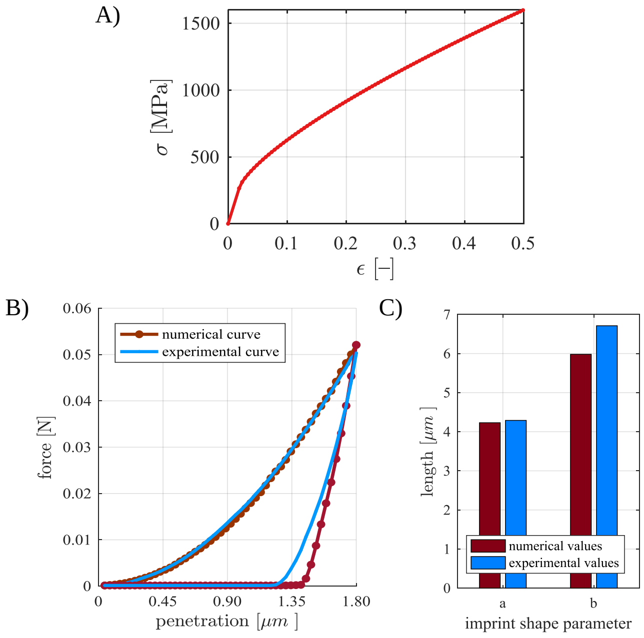

Figure 2: Constitutive law identified in the study (A), comparison between experimental and numerical counterparts: indentation curves (B) and imprint shape parameters (C)

Furthermore, it seems that to get more unique results of the inverse analysis the damage behavior should be included into the constitutive law. This statement is evidenced by the comparison of diagonal lengths of imprint (experimental vs. numerical). The length in numerical case is not very well reconstructed and should obtain better agreement with the experiment, see column *b* in Fig. 2C.

It is advised to use the Rockwell protocol with the conical indenter. The approach would greatly reduce the computation cost due to the axisymmetric FEM model.

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