Can sample-based approach outperform the classical dynamical anaysis? - experimental confirmation of the basin stability method

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Abstract

We show the first broad experimental confirmation of the basin stability approach. We study theoretically and experimentally the dynamics of a forced double pendulum. We examine the ranges of stability for nine different solutions of the system in a two parameter space, namely the amplitude and the frequency of excitation. We apply the path-following and the extended basin stability methods (Brzeski et. al., Meccanica 51(11), 2016) and we verify obtained theoretical results in experimental investigations. Comparison of the presented results show that the sample-based approach offers comparable precision to classical methods of analysis. However, it is much simpler to apply and can be used despite the type of dynamical system and its dimensions. Moreover, the sample-based approach has some unique advantages and can be applied without the precise knowledge of parameter values.

Keywords: Multistability, Basin stability, Monte Carlo, double pendulum

1. Introduction

There is a rich variety of different mathematical tools to analyze nonlinear dynamical systems. Still, more sophisticated methods are usually difficult to apply. For example, there are a number of different toolboxes that enable the path-following analysis but their functionality is strictly limited to the type of the investigated system and its dimensionality.

The dynamical analysis is especially challenging for multistable systems, where we have to consider multiple steady states that coexist in the phase space. It is a challenging problem and multistability is widely studied in many disciplines [1, 2, 3]. Therefore, also new tools to analyze multistable systems are being developed. In 2013 Menck et al. proposed a basin stability measure that uses Bernoulli trials to estimate the volume of a basin of attraction [4]. Despite it is new, the method was already successfully applied in numerous different applications [5, 4]. Recently, [6] we described an extended basin stability approach by taking into account mismatch in parameter values. This has practical fundamentals because all parameters values are measured or estimated with some finite precision and can slightly vary even during normal operation.

Now, we expand our approach and perform an analysis in a two parameter space. We compare the results from sample-based analysis with detailed two parameter bifurcation diagrams obtained using the path-following method. Finally, we confront both methods with experimental data that we use as a benchmark. This is done for 9 different periodic solutions that coexist in a notably wide range of the parameter values. The results enable us to critically compare the accuracy of both methods and show their strengths and weaknesses. Apart from that, we show that the sample-based approach can be applied without a precise knowledge of parameter values and it gives sensible results.

2. Model and methods

We consider the specific type of a double pendulum (see Fig. 1) which is a paradigmatic example in nonlinear dynamics. The first pendulum rod is mounted horizontally and connected to the base with a pin joint at one end and via a spring on the second end. Hence, it can only oscillate. The second pendulum is connected to the first one with a rotational pivot at the distance x_1 between both pin joints. The support is mounted on a shaker and excited kinematically in the vertical direction.

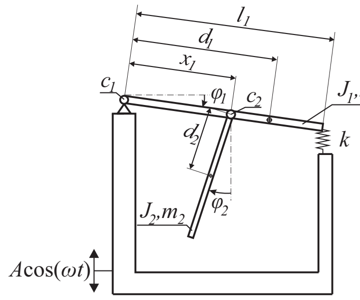

Figure 1: The physical model of the considered double pendulum excited kinematically with its parameters.

The behaviour of the system is described by the set of two second order oridinary differential equations. We analyze the dynamics of the above system using two different approaches: path-following method and sample-based method and validate them with experiments. As the controlling parameters we take the amplitude A and the frequency ω of the external excitation. The system is multistable, hence we observe a coexistence of solutions for fixed parameter values. The aim of our study is to investigate the ranges of attractors' stability in the (A, ω) plane. Results from both methods are then compared experimentaly obtained boundaries of stability in the two parameter space (A, ω).

We detected and further considered 9 different solution of the system nemely: $1:1$, $1:2$, $1:3$, $1:4$, $1:6$, $1:8$ oscillations, and $1:1$, $1:2$, $1:3$ rotations. For all 9 solutions we get the boundaries of stability using the above methods. To show all of the obtained results and ensure an easy comparison between the methods, we need to develop a clever presentation scheme. In Fig. 2 we present the results yielded for 1 : 2 oscillations to explain the presentation scheme that will be used. Arrows in Fig. 2 show how the data are interchanged between the panels.

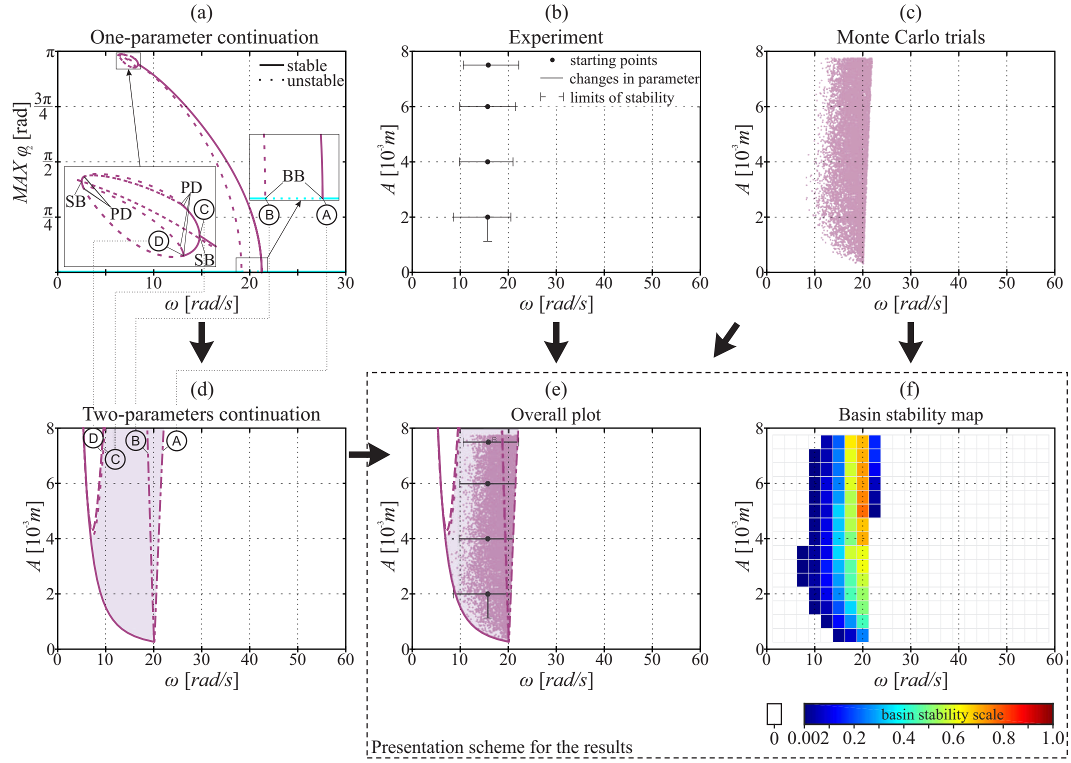

Figure 2: Presentation scheme for the stability analysis obtained using different approaches for 1 : 2 oscillatory solution. Panels (a,d) were obtained from the continuation of a periodic solution, in panel (b) we present experimental data and in panels (c,f) the results obtained with a sample-based method. In subplot (c) we compare the three approaches.

3. Conclusions

The aim of our investigations was to compare different analysis approaches and experimentally validate the accuracy of the sample-based method. We analysed 9 dufferent solutions that coexist in a wide range of parameter values.

Our results from both numerical methods are in a remarkably good agreement with the experimental data. Hence, we claim that the sample-based approach ensures the level of accuracy comparable with the classical path-following method. The advantage of the presented method is that it enables to analyse the influence of infinite number of parameters simultaneously, which is impossible for classical methods of analysis. Also, contrary to classical methods, the computational effort does not increase with the dimensions of the system. Moreover, the method enables to detect hidden attractors and solutions with rather meager basins of attraction. Apart from that, the method is straight forward and does not require any specialized knowledge.

The presented results are the first broad comparison between the path-following method, the basin stability approach and the experimental investigation. We show that the sample-based methods are a reliable tool for the analysis of complex dynamical systems. Moreover, we prove that the extended basin stability method has significant advantages which make it robust and appropriate for many applications in which classical analysis methods are difficult to apply.

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