Modelling of shape memory alloy oscillator and its application to middle ear structural reconstruction

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Abstract

The paper presents numerical research of a reconstructed middle ear system using the element made of shape memory alloy. Shape memory alloy is modelled based on hysteretic nonlinear theory. Bifurcation analysis of the system exhibits different kind of solutions starting from regular and ending with chaotic vibrations.

Keywords: middle ear mechanics, shape memory alloy

1. Introduction

Shape memory alloys (SMAs) belong to a group of structural materials, which are used in biomedical industry to built dental braces, stent grafts and in other engineering fields ranging from aerospace engineering and robotics. They are used to built actuators, elements of jet engines and so on. SMAs are materials, which exhibit a reversible thermo-elastic transformation between martensite and austenite influenced by the stress or temperature factor. When the alloys change their shape under the influence of temperature the phenomenon is called as shape memory effect (SME). SMEs can be divided into two types: one- or two-way. The one way shape memory effect retains a deformed state after the removal of an external force, and then recovers to its original shape upon heating. The two-way SME is similar to one-way SME but can also preserve its shape at both high and low temperatures. SMAs can subsist in two different phases with three different crystalline lattice structures: twinned martensite, detwinned martensite and austenite. In these configurations six possible transformations can occur. The austenite structure is stable at high temperatures, and the martensite structure is stable at lower temperatures. When SMA is heated, it begins to transform from the martensite into the austenite phase. The twinned martensite is stable when material is free from stress. The conversion from twinned to detwinned martensite follows the loading process. When the process of loading or unloading is finished, some residual strain remains, that is the reason of the reverse transformation from detwinned to twinned martensite, is not completed. Further deformation causes the transformation from detwinned martensite to austenite, then the SME can be observed . This process appears by heating of SMA. In order to regain from the structure of austenite to twinned martensite the SMA should be cooled to the temperature, at which the phase of martensite is stable. This process forms a hysteresis loop known as pseudoelastic effect which can be very complicated in modelling process.

The thermo-mechanical properties of SMAs can be modelled at different scales. In microscopic and mesoscopic approaches [5] the material behaviour is modelled starting from the molecular and lattice levels, respectively. Other class of models is based on a macroscopic approach, where only phenomenological features of the SMAs are taken into account [7]. Most often these models are based on an assumed phase transformation kinetics and consider certain mathematical functions to describe the phase transformation behaviour of the material. This approach was first proposed by Tanaka and Nagaki [13], and it provided a stimulus for the scientific community to present other modified transformation kinetics laws, see e.g. papers by Liang and Rogers [6] and Brinson [1]. These models are very popular in the literature, and play an important role in SMAs structures modelling and analysis [7].

Another group of phenomenological models is based on Devonshire's theory which postulates a free energy potential function expressed as a polynomial in material strain. Initially proposed for a one-dimensional stress state by Falk [2], it was later extended for a three-dimensional context by Falk and Konopka [3]. Afterwards, a similar model was proposed by Fremond [4] and many others [8, 9, 10], also in a simplified form. These free energy potential models can reproduce both the pseudoelastic and shape memory effects depending on the temperature and the stress-strain state. Therefore, the Falk's nonlinear SMA model is used in the present study.

The provided paper presents dynamics of a two degrees of freedom SMA oscillator. The model of SMA spring is based on the stiffness variation with respect to the temperature, strain and strain velocity [14]. The 2dof system is adopted to model a reconstructed human middle ear. The paper focuses on numerical analysis of system dynamics under various excitation conditions.

2. Model of a reconstructed middle ear

The presented model of a reconstructed middle ear consists of two lumped masses: the malleus (m_M) and the stapes (m_S) connected by a spring made of shape memory alloy (Fig.1). Moreover the masses are fixed to the base through springs (k) and dampers (c) which represent the ligaments of middle ear (AML and AL), the tympanic membrane (TM) and the cochlea (C).

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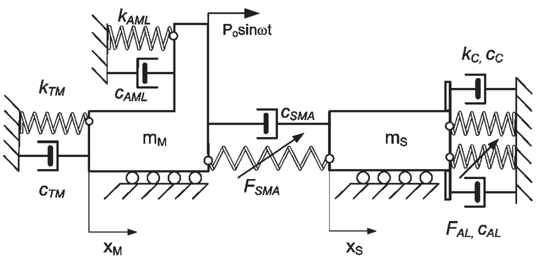

Figure 1: Two degree of freedom model with SMA element

The differential equation of the system motion can be transformed to the non-dimensional form:

$$\begin{aligned} &\ddot{x}_1 + \alpha_{11}x_1 - \alpha_2 x_2 + \delta_{11}\dot{x}_1 - \delta\dot{x}_2 + \alpha_{13}x_1^3 + \\ &\beta_2(x_1 - x_2)^2 + \beta_3(x_1 - x_2)^3 + \delta_3(\dot{x}_1 - \dot{x}_2)^3 \\ &\qquad = q\sin(\omega\tau) \\ &m\ddot{x}_2 - \alpha_2 x_1 + \alpha_{21}x_2 - \delta\dot{x}_1 + \delta_{21}\dot{x}_2 + \alpha_{22}x_2^2 + \\ &+\alpha_{23}x_2^3 - \beta_2(x_1 - x_2)^2 - \beta_3(x_1 - x_2)^3 - \\ &\qquad \delta_3(\dot{x}_1 - \dot{x}_2)^3 = 0 \end{aligned} \tag{1}$$

where, x_1 and x_2 are dimensionless coordinates of x_M and x_S. The coefficient α, β, δ and m are non-dimensional factors, q and ω are the amplitude and the frequency of the external excitation.

3. Numerical results

The presented model is analysed numerically to find dynamic responses on external excitation. The investigations are perform in Matlab-Simulink using Runge-Kutta method. Vibrations of the middle ear are presented in the form of bifurcation diagram (Fig.2) where the excitation amplitude (q) is changed. One can observe a variety of solutions starting from regular ones and ending at chaotic.

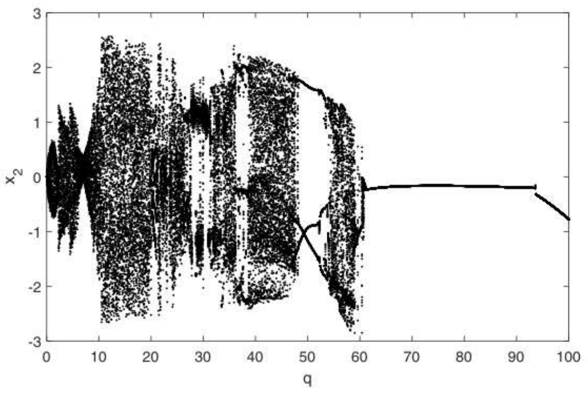

Figure 2: Bifurcation diagram of x_2 vibrations

4. Conclusions

The presented nonlinear model of 2dof oscillator with SMA spring exhibits rich dynamics which will be studied thoroughly in the full paper. However, some interesting symptoms of chaotic and regular vibrations are demonstrated in the tested range of excitation amplitudes.

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