Drgania własne belki swobodnie podpartej

$ORIGIN := 1$

$E := 10\ GPa \qquad b := 10\ cm \qquad h := 10\ cm \qquad L := 10\ m \qquad \rho := 600\ \dfrac{kg}{m^3}$

$A := b \cdot h \qquad J := \dfrac{A \cdot h^2}{12} \qquad \mu := \rho \cdot A \qquad c := \sqrt{\dfrac{E \cdot J}{\mu}}$

$A = 100\ cm^2 \qquad J = 833.333\ cm^4 \qquad \mu = 6\ \dfrac{kg}{m} \qquad c = 117.851\ \dfrac{m^2}{s} \qquad EJ := E \cdot J = 83.333333\ kN \cdot m^2$

Warunki brzegowe:

$y(0) = 0 \qquad y(L) = 0$

$M(0) = 0 \qquad M(L) = 0$

$M(x) = \dfrac{d^2}{dx^2} y(x)$

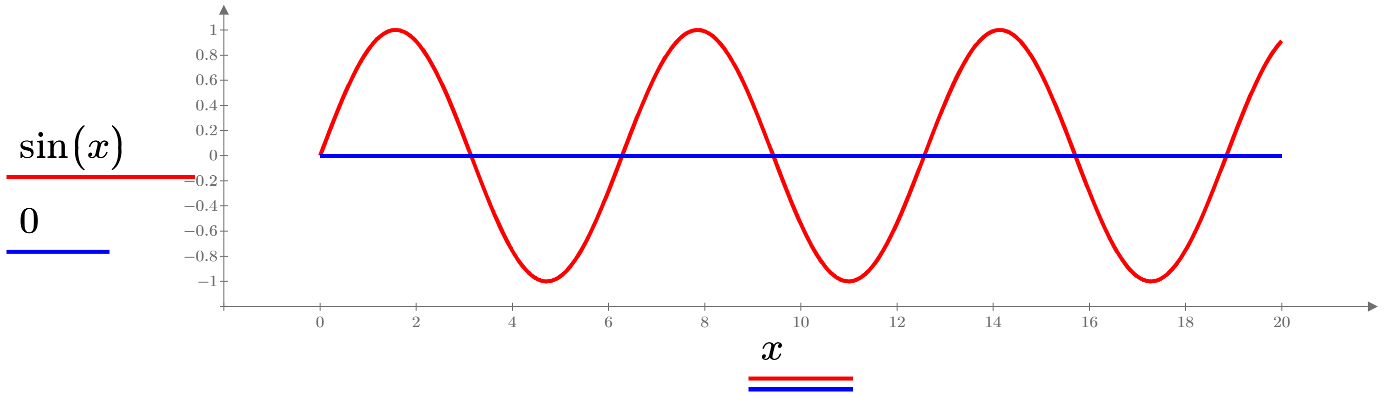
Równanie różniczkowe opisujące swobodne drgania (bez tłumienia) pręta prostego przy zastosowaniu modelu Bernoulliego (bez udziału sił poprzeczych i bezwładności obrotowej).

$$EJ \cdot \frac{d^4}{dx^4} w(x,t) + \mu \cdot \frac{d^2}{dt^2} w(x,t) = 0 \qquad (1)$$

Przyjmując rozwiązanie w postaci $w(x,t) = y(x) \cdot sin(\omega \cdot t)$ *otrzymamy:*

$$\left[\frac{d^4}{dx^4} y(x) - \frac{\mu \cdot \omega^2}{EJ} \cdot y(x)\right] \cdot sin(\omega \cdot t) = 0 \quad \text{--------->} \quad \frac{d^4}{dx^4} y(x) - \frac{\varphi^4}{L^4} \cdot y(x) = 0 \qquad \varphi = L \cdot \sqrt{\frac{\omega}{c}}$$

Równanie przestępne, wynikające z warunków brzegowych, dające częstości drgań własnych: $sin(\varphi) = 0$



$n := 8$

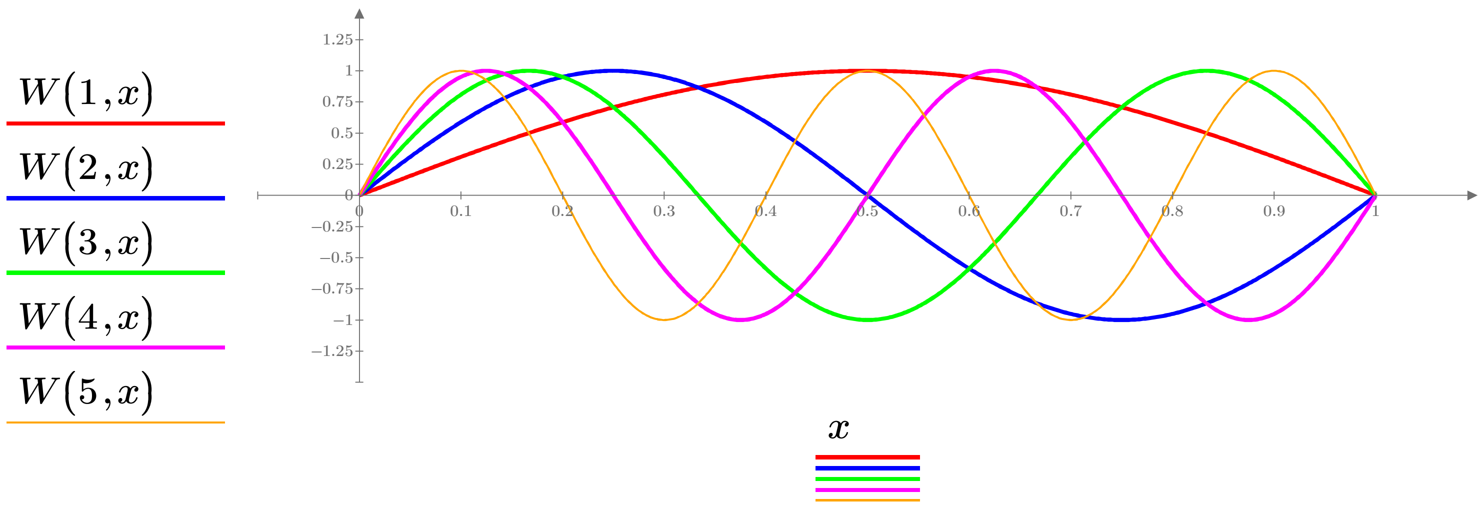
$i := 1..2\ n$ $\qquad \varphi_i := i \cdot \pi$ $\qquad \omega_A := \frac{c}{L^2} \cdot \varphi^2$

Postacie drgań własnych

$W(n, x) := \sin(\varphi_n \cdot x)$

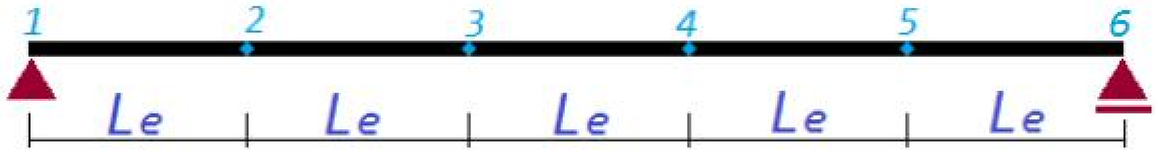
$$\varphi = \begin{bmatrix} 3.141593 \\ 6.283185 \\ 9.424778 \\ 12.566371 \\ 15.707963 \\ 18.849556 \\ 21.991149 \\ 25.132741 \\ 28.274334 \\ 31.415927 \\ 34.557519 \\ 37.699112 \\ \vdots \end{bmatrix}$$

$$\omega_A = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \vdots \\ 15 \end{array} \begin{bmatrix} 11.63144 \\ 46.525761 \\ 104.682963 \\ 186.103045 \\ 290.786008 \\ 418.731852 \\ \vdots \\ \end{bmatrix} \frac{rad}{s}$$



$$\frac{\omega_A}{2 \cdot \pi} = \begin{bmatrix} 1.851 \\ 7.405 \\ 16.661 \\ 29.619 \\ 46.28 \\ 66.643 \\ 90.709 \\ 118.477 \\ 149.947 \\ 185.12 \\ 223.995 \\ 266.573 \\ \vdots \end{bmatrix} \frac{1}{s}$$

Drgania własne belki obliczone metodą elementów skończonych



$l:=1\ m$ <--- *jednostka długości lub długość porównawcza*

$Le:=\dfrac{L}{n}$ $\kappa:=\dfrac{EJ}{Le^2}=53.333333\ kN$ $\lambda e:=\dfrac{Le}{l}=1.25$

$Ls:=2$ $Lw:=n+1$ *- Liczba węzłów* $Lr:=Ls\cdot Lw$ *- Liczba równań*

Funkcja DBM - Dodaj Blok Macierzy

$$DBM(A,B,w,k):=\left\| \begin{array}{l} \text{for } i\in 1..\text{rows}(B) \\ \quad \left\| \text{for } j\in 1..\text{cols}(B) \right. \\ \quad\quad \left\| A_{w+i-1,k+j-1}\leftarrow B_{i,j}+A_{w+i-1,k+j-1} \right. \\ A \end{array} \right.$$

Agregacja macierzy globalnych belki

$$Agrg\_B(N,Aa,Ab,Ac):=\left\| \begin{array}{l} Lss\leftarrow 2 \\ Lr\leftarrow Lss\ (N+1) \\ A_{Lr,Lr}\leftarrow 0 \\ \text{for } e\in 1..N \\ \quad \left\| \begin{array}{l} n\leftarrow e\cdot Lss-1 \\ k\leftarrow n+2 \\ A\leftarrow DBM(A,Aa,n,n) \\ A\leftarrow DBM(A,Ac,k,k) \\ A\leftarrow DBM(A,Ab,n,k) \\ A\leftarrow DBM(A,Ab^{T},k,n) \end{array} \right. \\ A \end{array} \right.$$

$$Ae=\begin{bmatrix} Aa & Ab \\ Ab^{T} & Ac \end{bmatrix}$$

$\omega_0:=\dfrac{20}{s}$ $p_{Lr}:=0$ $p_7:=1$ $p_{13}:=-0.8$

$$P1(t):=\left\| \begin{array}{l} S\leftarrow 0\cdot kN \\ \text{if } t<\dfrac{\pi}{\omega_0} \\ \quad \left\| S\leftarrow 1\ kN\cdot\sin(\omega_0\cdot t) \right. \\ S \end{array} \right.$$

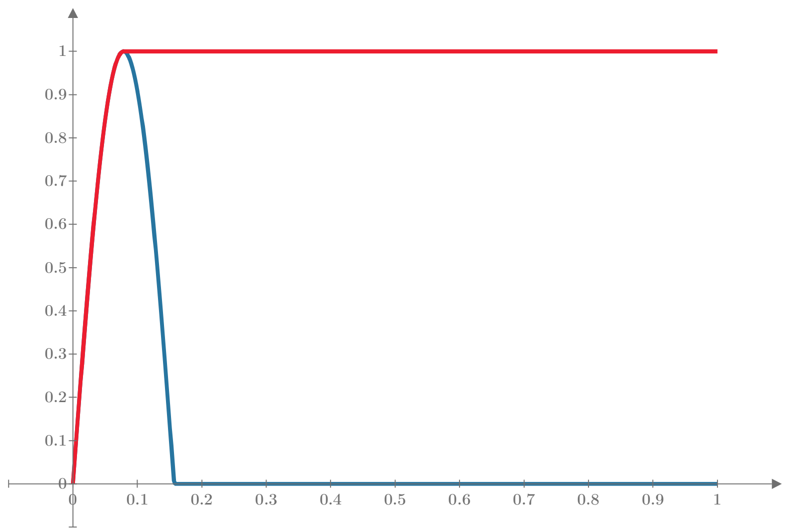
$$P2(t):=\left\| \begin{array}{l} S\leftarrow 1\cdot kN \\ \text{if } t<\dfrac{\pi}{2\ \omega_0} \\ \quad \left\| S\leftarrow 1\ kN\cdot\sin(\omega_0\cdot t) \right. \\ S \end{array} \right.$$

$p1(t):=p\cdot P1(t)$

$P1(0.2\ s)=0\ kN$

$p2(t):=p\cdot P2(t)$

$P2(0.2\ s)=1\ kN$



$P1(t\cdot 1\ s)\ (kN)$

$P2(t\cdot 1\ s)\ (kN)$

t

Macierz sztywności elementów belki

$$K=\frac{EJ}{Le^2}\cdot\begin{bmatrix}\frac{12}{Le} & 6 & \frac{-12}{Le} & 6\\ 6 & 4\cdot Le & -6 & 2\cdot Le\\ \frac{-12}{Le} & -6 & \frac{12}{Le} & -6\\ 6 & 2\cdot Le & -6 & 4\cdot Le\end{bmatrix}$$

$$K=\kappa\cdot\begin{bmatrix}Ka & Kb\\ Kb^{T} & Kc\end{bmatrix}$$

$$\kappa=53.333333\ kN$$

Bloki bezwymiarowej macierzy sztywności elementu

$$Ka:=\begin{bmatrix}\frac{12}{\lambda e} & 6\\ 6 & 4\cdot\lambda e\end{bmatrix}\qquad Kb:=\begin{bmatrix}\frac{-12}{\lambda e} & 6\\ -6 & 2\cdot\lambda e\end{bmatrix}\qquad Kc:=\begin{bmatrix}\frac{12}{\lambda e} & -6\\ -6 & 4\cdot\lambda e\end{bmatrix}$$

Agregacja globalnej bezwymiarowej macierzy sztywności belki

$$K:=Agrg\_B(n,Ka,Kb,Kc)$$

$$K=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 6 & 5 & -6 & 2.5 & 0 & 0 & 0 & \\ -9.6 & -6 & 19.2 & 0 & -9.6 & 6 & 0 & \\ 6 & 2.5 & 0 & 10 & -6 & 2.5 & 0 & \\ 0 & 0 & -9.6 & -6 & 19.2 & 0 & -9.6 & \\ 0 & 0 & 6 & 2.5 & 0 & 10 & -6 & \\ 0 & 0 & 0 & 0 & -9.6 & -6 & 19.2 & \\ 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & -9.6 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ & & & & & & & \ddots\end{bmatrix}$$

$$p_{F_{13}}:=-0.8\ kN$$

$$p_{F_{Lr}}:=0\ kN\qquad p_{F_5}:=1\ kN\qquad f_F:=\frac{p_F}{\kappa}$$

$$i:=1..Lr$$

$$w:=1\qquad K_{w,i}:=0\qquad K_{w,w}:=1$$

$$w:=Lr-1\qquad K_{w,i}:=0\qquad K_{w,w}:=1$$

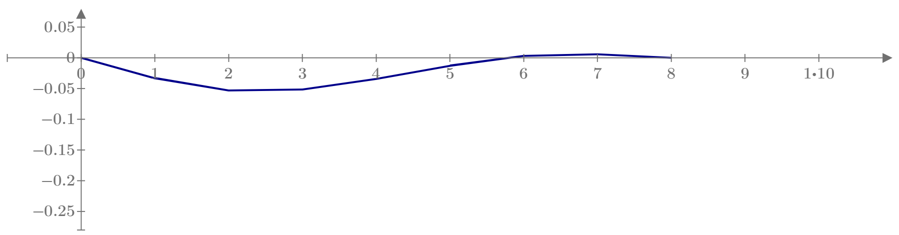
$$y:=\text{lsolve}(K,f_F)$$

$$i:=1..\frac{Lr}{2}$$

$$v_i:=y_{2\cdot i-1}$$

$$y=\begin{bmatrix}0\\0.028125\\0.0330078\\0.0229688\\0.053125\\0.0075\\0.0513672\\-0.0089063\\0.034375\\-0.016875\\0.0126953\\-0.0164063\\-0.003125\\-0.0075\\-0.0056641\\0.0023437\\0\\0.005625\end{bmatrix}\qquad p_F=\begin{bmatrix}0\\0\\0\\0\\1\\0\\0\\0\\0\\0\\0\\0\\\vdots\end{bmatrix}kN$$

$$v=\begin{bmatrix}-2.891206\cdot10^{-17}\\3.300781\cdot10^{-2}\\5.3125\cdot10^{-2}\\5.136719\cdot10^{-2}\\3.4375\cdot10^{-2}\\1.269531\cdot10^{-2}\\-3.125\cdot10^{-3}\\-5.664062\cdot10^{-3}\\0\end{bmatrix}$$



$-v_i$

$i-1$

$$\frac{1\ kN\cdot L^3}{48\cdot EJ}=25\ cm$$

$$u_{max}:=\max(y)=0.053125$$

Diagonalna macierz bezwładności elementów belki

$$M = \frac{\mu \cdot Le^2}{24} \cdot \begin{bmatrix} \frac{12}{Le} & 0 & 0 & 0 \\ 0 & Le & 0 & 0 \\ 0 & 0 & \frac{12}{Le} & 0 \\ 0 & 0 & 0 & Le \end{bmatrix}$$

$$M = \eta \cdot \begin{bmatrix} Ma & 0 \\ 0 & Ma \end{bmatrix}$$

$$\eta := \frac{\mu \cdot Le^2}{24} = \left(3.90625 \cdot 10^{-1}\right) \, \boldsymbol{kg \cdot m} \qquad Ma := \begin{bmatrix} \frac{12}{\lambda e} & 0 \\ 0 & \lambda e \end{bmatrix} \qquad M0 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\theta := \frac{\eta}{\kappa} = \left(7.324219 \cdot 10^{-6}\right) \, \boldsymbol{s}^2$$

Aby zmniejszyć wpływ bezwładności obrotowej można przyjąć Ma ze współczynnikiem 0<a<1

$$a := 0.1 \qquad Ma := \begin{bmatrix} \frac{12}{\lambda e} & 0 \\ 0 & a \cdot \lambda e \end{bmatrix}$$

Agregacja diagonalnej bezwymiarowej macierzy bezwładności

$$M := Agrg\_B(n, Ma, M0, Ma)$$

$$M = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 19.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 19.2 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 19.2 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.2 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.2 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 & & & & & & & & & & & \ddots
\end{bmatrix}$$

Uwzględnienie warunków brzegowych

$$i := 1 .. Lr$$

$$w := 1 \qquad K_{w,i} := 0 \qquad K_{i,w} := 0 \qquad K_{w,w} := 1 \qquad M_{w,i} := 0 \qquad M_{w,w} := 1$$

$$w := Lr - 1 \qquad K_{w,i} := 0 \qquad K_{i,w} := 0 \qquad K_{w,w} := 1 \qquad M_{w,i} := 0 \qquad M_{w,w} := 1$$

$$\kappa \cdot \left[K - \omega^2 \; \theta \cdot M\right] \, y = 0 \qquad \left|M^{-1} \cdot K - \sigma \cdot I\right| = 0 \qquad \omega^2 \cdot \theta = \sigma \qquad |M| = 9.172943 \cdot 10^2$$

$$\sigma := \text{eigenvals}\left(M^{-1} \cdot K\right) \qquad \omega := \text{sort}\left(\sqrt{\frac{\sigma}{\theta}}\right)$$

$$\omega^{\mathrm{T}} = [11.62 \;\; 46.39 \;\; 103.91 \;\; 183.03 \;\; 280.6 \;\; 369.5 \;\; 369.5 \;\; 387.75 \;\; 482.09 \;\; 1652.47 \;\; 1723.02 \;\; 1900.84 \;\; 2127.53 \;\; 2358.89 \;\; \ldots] \; \frac{\boldsymbol{rad}}{\boldsymbol{s}}$$

Równanie wymuszonych, tłumionych drgań belki

$$\psi_i = \frac{u_i}{l}$$

e

i ||===============|| j

<--------- Le ---------->

$$y_e = \begin{bmatrix} \psi_i \\ \varphi_i \\ \psi_j \\ \varphi_j \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \therefore \\ y_N \end{bmatrix}$$

$$K_@ \cdot u(t) + C_@ \cdot u'(t) + M_@ \cdot u''(t) = p(t) \qquad u' = \frac{d}{dt} u$$

$C_@ = \alpha \cdot M_@ + \beta \cdot K_@$ *<--- macierz tłumienia Rayleigha*

$$\begin{bmatrix} \frac{1}{\omega_1} & \omega_1 \\ \frac{1}{\omega_2} & \omega_2 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 2 \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$$

Współczynniki (a,b) macierzy tłumienia

$$C_@ = \alpha \cdot \eta \cdot M + \beta \cdot \kappa \cdot K$$

$$\kappa \cdot K \cdot y(t) + C_@ \cdot y'(t) + \eta \cdot M \cdot y''(t) = p(t)$$

$$K \cdot y(t) + \beta \cdot C \cdot y'(t) + \theta \cdot M \cdot y''(t) = f(t)$$

$$\omega_1 := \omega_{A_1} \qquad \omega_2 := \omega_{A_2} \qquad \zeta_1 := 0.05 \qquad \zeta_2 := 0.07$$

$$\kappa = 53.3 \ \mathbf{kN} \qquad \eta = 0.391 \ \mathbf{N} \cdot \mathbf{s}^2 \qquad \theta := \frac{\eta}{\kappa} = \left(7.324219 \cdot 10^{-6}\right) \mathbf{s}^2$$

$$\beta := 2 \frac{\omega_1 \cdot \zeta_1 - \omega_2 \cdot \zeta_2}{\omega_1^{\ 2} - \omega_2^{\ 2}} \qquad \beta = \left(2.636532 \cdot 10^{-3}\right) \mathbf{s}$$

Macierze K, C, M, y, f - są bezwymiarowe

$$C := K + \frac{\alpha \cdot \theta}{\beta} \cdot M \qquad f(t) := \frac{1}{\kappa} \cdot p1(t)$$

$$\alpha := \omega_1 \cdot \left(2 \cdot \zeta_1 - \omega_1 \cdot \beta\right) \qquad \alpha = \left(8.064465 \cdot 10^{-1}\right) \frac{1}{\mathbf{s}}$$

$$C = \begin{bmatrix}
1.002 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 5 & -6 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & \\
 & & & & & & & & & & & & & & \ddots
\end{bmatrix}$$

Różnice centralne. *Chwile* (t-Δt), t, (t+Δt) *oznaczane są indeksami górnymi* ----->

$$y'_t = \frac{y^{t+\Delta t} - y^{t-\Delta t}}{2\ \Delta t} \qquad y''_t = \frac{y^{t+\Delta t} - 2\ y^t + y^{t-\Delta t}}{\Delta t^2}$$

Po podstawieniu różnic zamiast pochodnych otrzymamy:

$$t_{kr} := \frac{2}{\omega_{Lr-1}} = 0.000707\ s \qquad \Delta t := 0.0005\ s \qquad \Delta t < t_{kr} \qquad \frac{\Delta t}{t_{kr}} = 0.707$$

$$K \cdot y^t + \frac{\beta}{2\ \Delta t} \cdot C \cdot \left(y^{t+\Delta t} - y^{t-\Delta t}\right) + \frac{\theta}{(\Delta t)^2} \cdot M \cdot \left(y^{t+\Delta t} - 2\ y^t + y^{t-\Delta t}\right) = f^t$$

$$a_1 := \frac{(\Delta t)^2}{\theta} = 3.413333 \cdot 10^{-2} \qquad a_2 := \frac{\beta \cdot \Delta t}{2 \cdot \theta} = 8.999363 \cdot 10^{-2}$$

$$a_1 \cdot K \cdot y^t + a_2 \cdot C \cdot \left(y^{t+\Delta t} - y^{t-\Delta t}\right) + M \cdot \left(y^{t+\Delta t} - 2\ y^t + y^{t-\Delta t}\right) = a_1 \cdot f^t$$

To równanie można względem czasu scałkować metodą "explicit":

$$y^{t+\Delta t} \cdot \left(M + a_2 \cdot C\right) = a_1 \cdot f^t - a_1 \cdot K \cdot y^t + a_2 \cdot C \cdot y^{t-\Delta t} + M \cdot \left(2 \cdot y^t - y^{t-\Delta t}\right)$$

$$MC := M + a_2 \cdot C \qquad |MC| = 986733181.418 \qquad Mc1 := MC^{-1}$$

$$y^{t+\Delta t} = Mc1 \cdot \left[a_1 \cdot f^t - a_1 \cdot K \cdot y^t + a_2 \cdot C \cdot y^{t-\Delta t} + M \cdot \left(2 \cdot y^t - y^{t-\Delta t}\right)\right]$$

$$M_K := Mc1 \cdot \left(2 \cdot M - a_1 \cdot K\right) \qquad M_C := Mc1 \cdot \left(a_2 \cdot C - M\right)$$

$$y^{t+\Delta t} = r_f^{\ t} + M_K \cdot y^t + M_C \cdot y^{t-\Delta t} \qquad r(t) := Mc1 \cdot a_1 \cdot f(t)$$

Uwzględnienie warunków brzegowych

$i := 1..Lr$

$w := 1 \qquad M_{K_{w,i}} := 0 \qquad M_{K_{w,w}} := 1 \qquad M_{C_{w,i}} := 0 \qquad M_{C_{w,w}} := 1$

$w := Lr - 1 \qquad M_{K_{w,i}} := 0 \qquad M_{K_{w,w}} := 1 \qquad M_{C_{w,i}} := 0 \qquad M_{C_{w,w}} := 1$

$$\mathrm{Round}\left(\frac{11 \cdot \pi}{\omega_1 \cdot \Delta t}, 1\right) = 5946$$

$Nst := 5000$ *<---- liczba kroków czasowych* $\qquad \Delta t = 0.0005\ s$

$Y_{Lr,2} := 0$ *<---- inicjacja macierzy rozwiazania i warunków brzegowych*

$$y^{t+\Delta t} = r_f^{\ t} + M_K \cdot y^t + M_C \cdot y^{t-\Delta t}$$

$$Explicit\_Central\_Time(A, N, \Delta) := \left\| \begin{array}{l} \text{for } i \in 1..N \\ \quad \left\| \begin{array}{l} k \leftarrow \mathrm{cols}(A) \\ t \leftarrow (k-1) \cdot \Delta \\ y2 \leftarrow r(t) + M_K \cdot A^{\langle k \rangle} + M_C \cdot A^{\langle k-1 \rangle} \\ A \leftarrow \mathrm{augment}(A, y2) \end{array} \right. \\ A \end{array} \right.$$

$Y := Explicit\_Central\_Time(Y, Nst, \Delta t)$ $\qquad$ $i := 1..Nst$ $\qquad$ $t_i := (i-1) \cdot \Delta t$

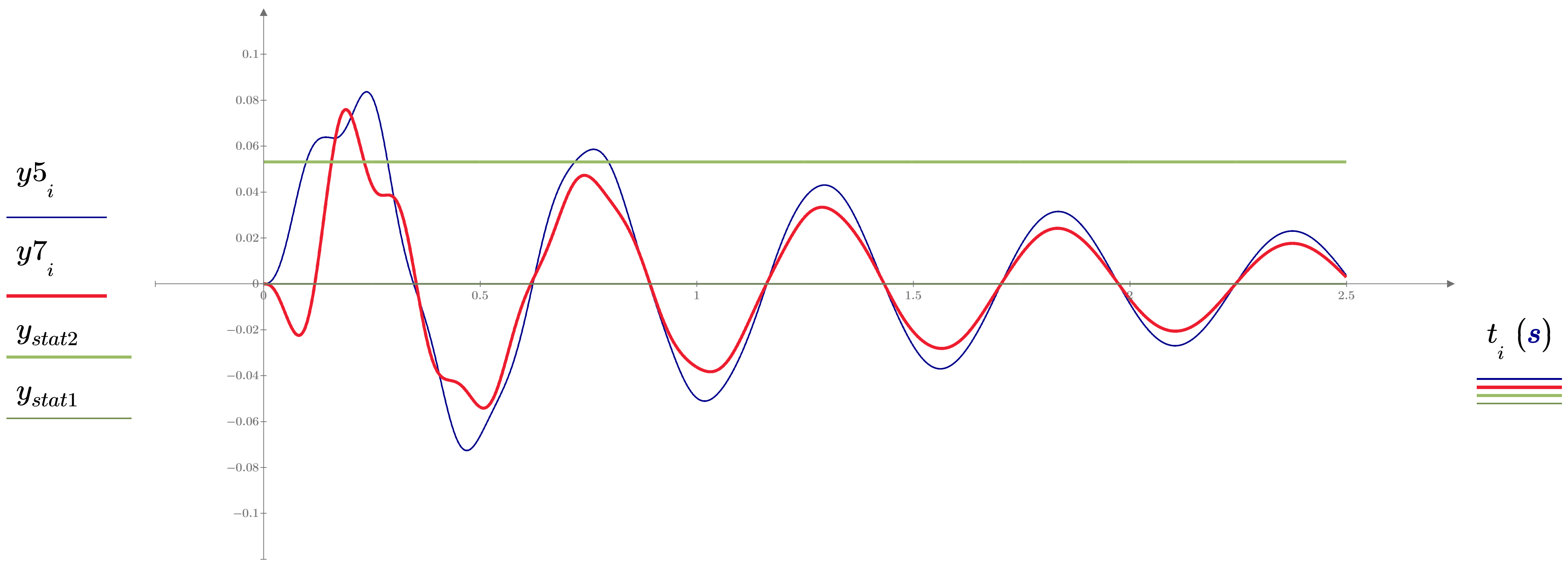
$t^T =$

0 …	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	… 4999	
…	1.04	1.0405	1.041	1.0415	1.042	1.0425	1.043	1.0435	1.044	1.0445	1.045	1.0455	1.046	1.0465	1.047	1.0475	1.048	1.0485	1.049	1.0495	…	s

$Y =$

	0 …	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	… 5001
0	…	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1		−0.018	−0.018	−0.018	−0.018	−0.018	−0.018	−0.018	−0.018	−0.018	−0.018	−0.018	−0.017	−0.017	−0.017	−0.017	−0.017	−0.017	−0.017	−0.017	
2		−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	−0.021	
3		−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	−0.016	
4		−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	−0.039	
5		−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	−0.012	
6		−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	−0.051	
7		−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006	
8		−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	−0.055	
9		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
10		−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	
11		0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	
12		−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	−0.038	
13		0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	
⋮ 17																					⋱

$y5_i := Y_{7,i}$ $\qquad$ $y7_i := Y_{13,i}$ $\qquad$ $y_{stat2} := u_{max} = 0.053125$ $\quad$ $y_{stat1} := 0$

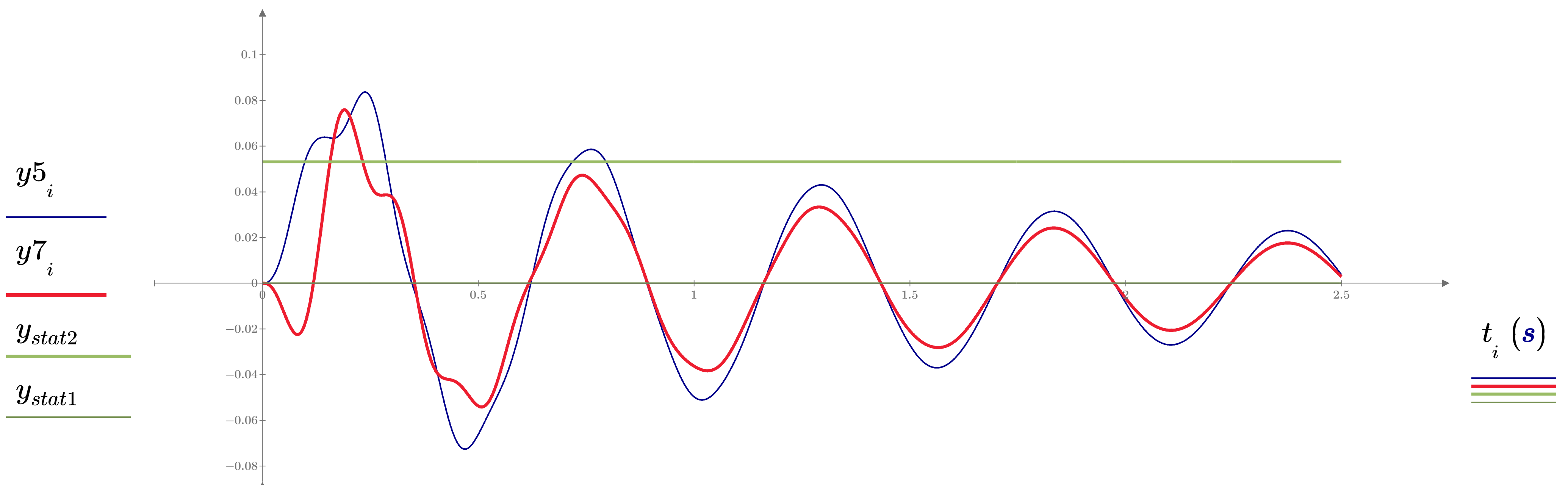


$\max(y5) = 0.083658$ $\qquad$ $y_{inf} := y7_{Nst} = 0.002898$

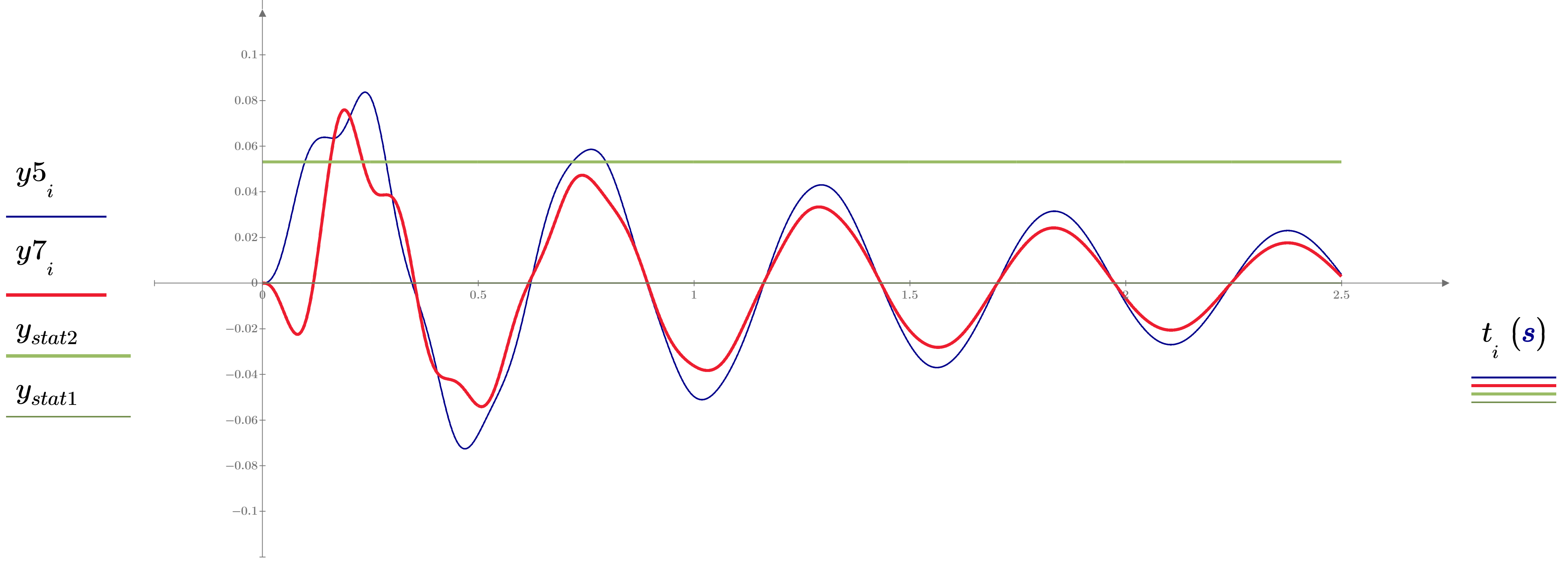
$$\frac{y_{inf}}{y_{stat2}} = 0.054552$$

$$\eta_{dyn} := \frac{\max(y7)}{y_{stat2}} = 1.429286$$

$$\eta_{dyn2} := 1.715522$$



“Newmark”



“Central difference”