Drgania własne belki swobodnie podpartej

$\mathrm{ORIGIN} := 1$

$E := 10\ GPa$ $\quad b := 10\ cm$ $\quad h := 10\ cm$ $\quad L := 10\ m$ $\quad \rho := 600\ \dfrac{kg}{m^3}$

$A := b \cdot h$ $\quad J := \dfrac{A \cdot h^2}{12}$ $\quad \mu := \rho \cdot A$ $\quad c := \sqrt{\dfrac{E \cdot J}{\mu}}$

$A = 100\ cm^2$ $\quad J = 833.333\ cm^4$ $\quad \mu = 6\ \dfrac{kg}{m}$ $\quad c = 117.851\ \dfrac{m^2}{s}$ $\quad EJ := E \cdot J = 83.333333\ kN \cdot m^2$

Warunki brzegowe: $\quad y(0) = 0 \quad y(L) = 0$

$M(0) = 0 \quad M(L) = 0$

Równanie różniczkowe opisujące swobodne drgania (bez tłumienia) pręta prostego przy zastosowaniu modelu Bernoulliego (bez udziału sił poprzeczych i bezwładności obrotowej).

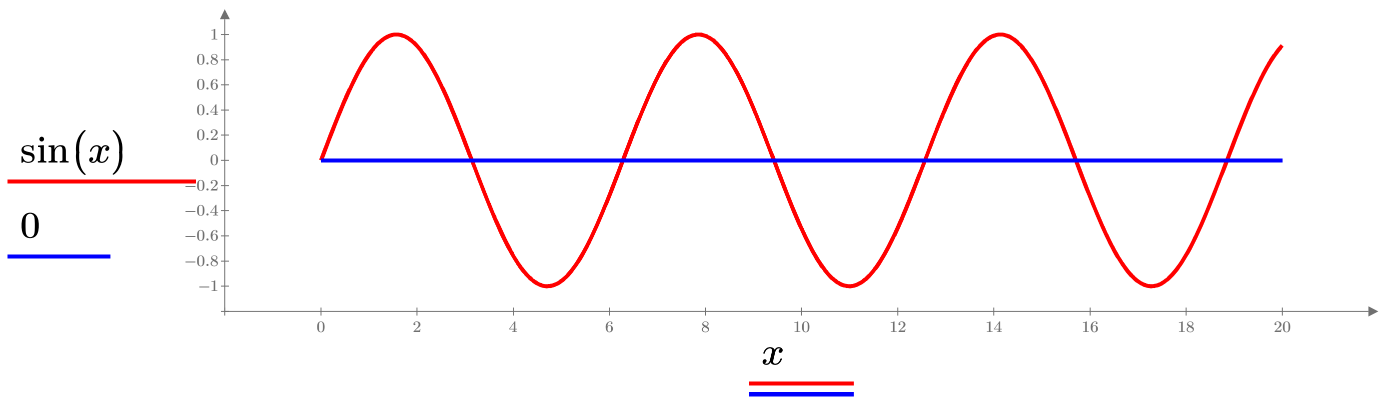
$$\frac{M(x)}{EJ} = \frac{d^2}{dx^2} y(x)$$

$$EJ \cdot \frac{d^4}{dx^4} w(x,t) + \mu \cdot \frac{d^2}{dt^2} w(x,t) = 0 \qquad (1)$$

Przyjmując rozwiązanie w postaci $\quad w(x,t) = y(x) \cdot sin(\omega \cdot t)$ $\quad$ *otrzymamy:*

$$\left[\frac{d^4}{dx^4} y(x) - \frac{\mu \cdot \omega^2}{EJ} \cdot y(x)\right] \cdot sin(\omega \cdot t) = 0 \quad \text{--------->} \quad \frac{d^4}{dx^4} y(x) - \frac{\varphi^4}{L^4} \cdot y(x) = 0 \qquad \varphi = L \cdot \sqrt{\frac{\omega}{c}}$$

Równanie przestępne, wynikające z warunków brzegowych, dające częstości drgań własnych: $sin(\varphi)=0$



$n:=8$

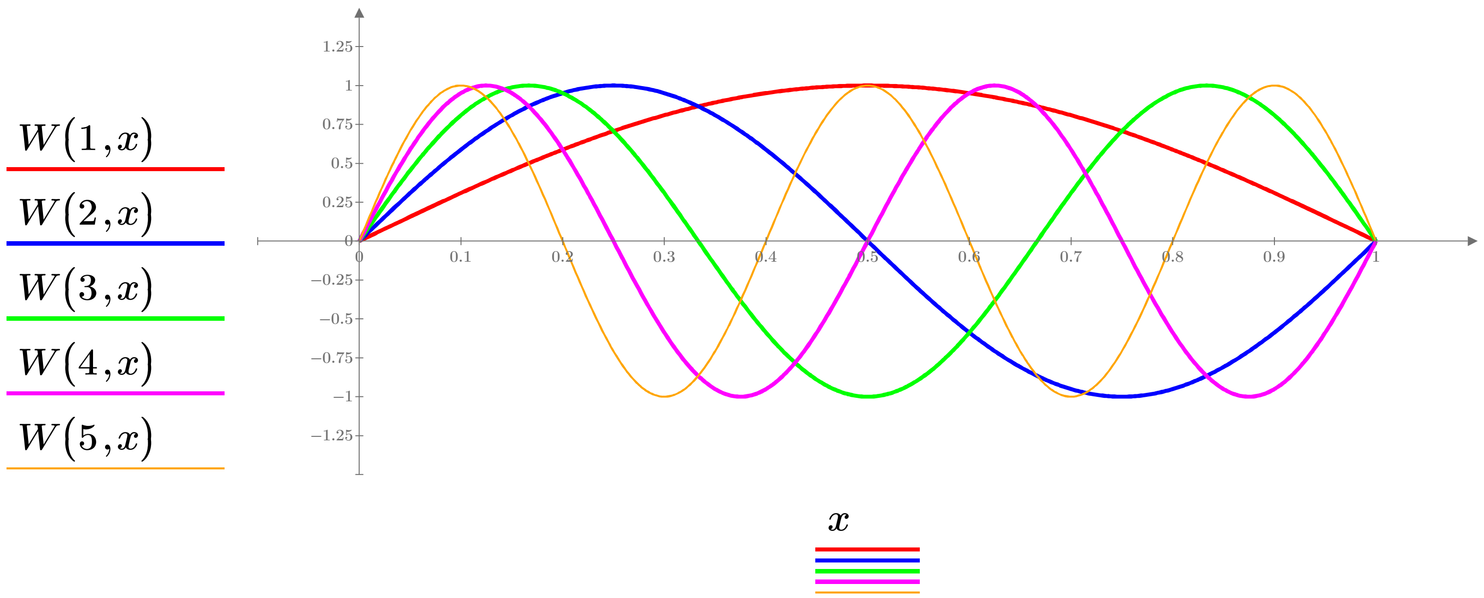
$i:=1..2\ n$ $\qquad \varphi_i:=i\cdot\pi$ $\qquad \omega_A:=\dfrac{c}{L^2}\cdot\varphi^2$

$$\varphi=\begin{bmatrix}3.141593\\6.283185\\9.424778\\12.566371\\15.707963\\18.849556\\21.991149\\25.132741\\28.274334\\31.415927\\34.557519\\37.699112\\\vdots\end{bmatrix}\qquad \frac{\omega_A}{2\cdot\pi}=\begin{bmatrix}1.851\\7.405\\16.661\\29.619\\46.28\\66.643\\90.709\\118.477\\149.947\\185.12\\223.995\\266.573\\\vdots\end{bmatrix}\frac{1}{s}$$

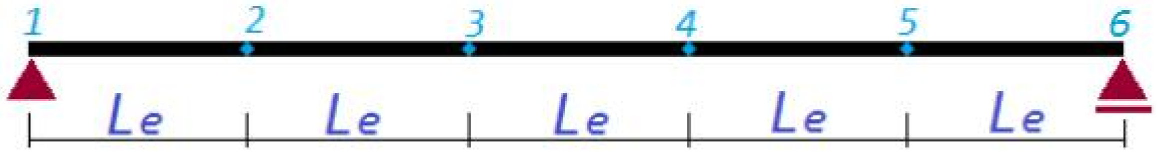
Postacie drgań własnych

$W(n,x):=\sin\left(\varphi_n\cdot x\right)$

$$\omega_A=\begin{bmatrix}11.6314\\46.5258\\104.683\\186.103\\290.786\\418.7319\\569.9406\\\vdots\end{bmatrix}\frac{rad}{s}$$



Drgania własne belki obliczone metodą elementów skończonych



$l := 1\ m$ <--- jednostka długości lub długość porównawcza

$Le := \frac{L}{n}$ $\kappa := \frac{EJ}{Le^2} = 53.333333\ kN$ $\lambda e := \frac{Le}{l} = 1.25$

$Ls := 2$ $Lw := n + 1$ - Liczba węzłów $Lr := Ls \cdot Lw$ - Liczba równań

Funkcja DBM - Dodaj Blok Macierzy

$$DBM(A,B,w,k) := \left\| \begin{array}{l} \text{for } i \in 1..\text{rows}(B) \\ \quad \left\| \text{for } j \in 1..\text{cols}(B) \right. \\ \qquad \left\| A_{w+i-1,k+j-1} \leftarrow B_{i,j} + A_{w+i-1,k+j-1} \right. \\ A \end{array} \right.$$

Agregacja macierzy globalnych belki

$$Agrg\_B(N,Aa,Ab,Ac) := \left\| \begin{array}{l} Lss \leftarrow 2 \\ Lr \leftarrow Lss\ (N+1) \\ A_{Lr,Lr} \leftarrow 0 \\ \text{for } e \in 1..N \\ \quad \left\| \begin{array}{l} n \leftarrow e \cdot Lss - 1 \\ k \leftarrow n + 2 \\ A \leftarrow DBM(A,Aa,n,n) \\ A \leftarrow DBM(A,Ac,k,k) \\ A \leftarrow DBM(A,Ab,n,k) \\ A \leftarrow DBM(A,Ab^{T},k,n) \end{array} \right. \\ A \end{array} \right.$$

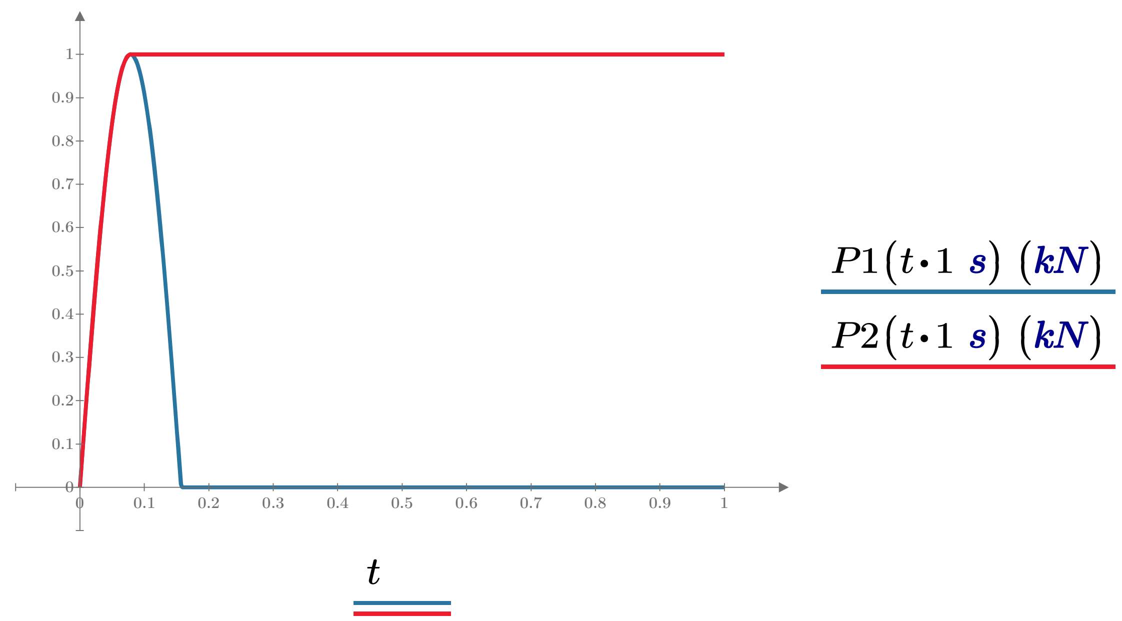
$$Ae = \begin{bmatrix} Aa & Ab \\ Ab^{T} & Ac \end{bmatrix}$$

$\omega_0 := \frac{20}{s}$ $p_{Lr} := 0$ $p_7 := 1$ $p_{13} := -0.8$

$$P1(t) := \left\| \begin{array}{l} S \leftarrow 0 \cdot kN \\ \text{if } t < \frac{\pi}{\omega_0} \\ \quad \left\| S \leftarrow 1\ kN \cdot \sin(\omega_0 \cdot t) \right. \\ S \end{array} \right. \qquad P2(t) := \left\| \begin{array}{l} S \leftarrow 1 \cdot kN \\ \text{if } t < \frac{\pi}{2\ \omega_0} \\ \quad \left\| S \leftarrow 1\ kN \cdot \sin(\omega_0 \cdot t) \right. \\ S \end{array} \right.$$

$p1(t) := p \cdot P1(t)$ $p2(t) := p \cdot P2(t)$

$P1(0.2\ s) = 0\ kN$ $P2(0.2\ s) = 1\ kN$



Macierz sztywności elementów belki

$$K = \frac{EJ}{Le^2}\cdot\begin{bmatrix} \frac{12}{Le} & 6 & \frac{-12}{Le} & 6 \\ 6 & 4\cdot Le & -6 & 2\cdot Le \\ \frac{-12}{Le} & -6 & \frac{12}{Le} & -6 \\ 6 & 2\cdot Le & -6 & 4\cdot Le \end{bmatrix}$$

$$K = \kappa\cdot\begin{bmatrix} Ka & Kb \\ Kb^{T} & Kc \end{bmatrix}$$

$$\kappa = 53.333333\ kN$$

Bloki bezwymiarowej macierzy sztywności elementu

$$Ka := \begin{bmatrix} \frac{12}{\lambda e} & 6 \\ 6 & 4\cdot\lambda e \end{bmatrix} \qquad Kb := \begin{bmatrix} \frac{-12}{\lambda e} & 6 \\ -6 & 2\cdot\lambda e \end{bmatrix} \qquad Kc := \begin{bmatrix} \frac{12}{\lambda e} & -6 \\ -6 & 4\cdot\lambda e \end{bmatrix}$$

Agregacja globalnej bezwymiarowej macierzy sztywności belki

$$K := Agrg\_B(n, Ka, Kb, Kc)$$

$$K = \begin{array}{c|c} & \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & \cdots 17 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ \vdots \\ 17 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 6 & 5 & -6 & 2.5 & 0 & 0 & 0 & \\ -9.6 & -6 & 19.2 & 0 & -9.6 & 6 & 0 & \\ 6 & 2.5 & 0 & 10 & -6 & 2.5 & 0 & \\ 0 & 0 & -9.6 & -6 & 19.2 & 0 & -9.6 & \\ 0 & 0 & 6 & 2.5 & 0 & 10 & -6 & \\ 0 & 0 & 0 & 0 & -9.6 & -6 & 19.2 & \\ 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & -9.6 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ & & & & & & & \ddots \end{bmatrix} \end{array}$$

$$p_{F_{13}} := -0.8\ kN$$

$$p_{F_{Lr}} := 0\ kN \qquad p_{F_5} := 1\ kN \qquad f_F := \frac{p_F}{\kappa}$$

$$i := 1 \,..\, Lr$$

$$w := 1 \qquad K_{w,i} := 0 \qquad K_{w,w} := 1$$

$$w := Lr - 1 \qquad K_{w,i} := 0 \qquad K_{w,w} := 1$$

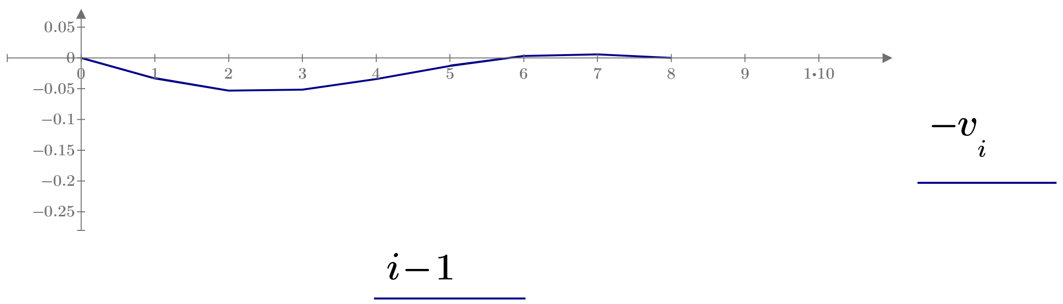
$$y := \text{lsolve}(K, f_F)$$

$$i := 1 \,..\, \frac{Lr}{2}$$

$$v_i := y_{2\cdot i-1}$$

$$y = \begin{bmatrix} 0 \\ 0.028125 \\ 0.0330078 \\ 0.0229688 \\ 0.053125 \\ 0.0075 \\ 0.0513672 \\ -0.0089063 \\ 0.034375 \\ -0.016875 \\ 0.0126953 \\ -0.0164063 \\ -0.003125 \\ -0.0075 \\ -0.0056641 \\ 0.0023437 \\ 0 \\ 0.005625 \end{bmatrix} \qquad p_F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} kN$$

$$v = \begin{bmatrix} -2.891206\cdot10^{-17} \\ 3.300781\cdot10^{-2} \\ 5.3125\cdot10^{-2} \\ 5.136719\cdot10^{-2} \\ 3.4375\cdot10^{-2} \\ 1.269531\cdot10^{-2} \\ -3.125\cdot10^{-3} \\ -5.664062\cdot10^{-3} \\ 0 \end{bmatrix}$$



$$\frac{1\ kN\cdot L^3}{48\cdot EJ} = 25\ cm$$

$$u_{max} := \max(y) = 0.053125$$

Diagonalna macierz bezwładności elementów belki

$$M=\frac{\mu\cdot Le^2}{24}\cdot\begin{bmatrix}\frac{12}{Le} & 0 & 0 & 0\\ 0 & Le & 0 & 0\\ 0 & 0 & \frac{12}{Le} & 0\\ 0 & 0 & 0 & Le\end{bmatrix}$$

$$M=\eta\cdot\begin{bmatrix}Ma & 0\\ 0 & Ma\end{bmatrix}$$

$$\eta:=\frac{\mu\cdot Le^2}{24}=\left(3.90625\cdot10^{-1}\right)\ \boldsymbol{kg\cdot m}\qquad Ma:=\begin{bmatrix}\frac{12}{\lambda e} & 0\\ 0 & \lambda e\end{bmatrix}\qquad M0:=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\theta:=\frac{\eta}{\kappa}=\left(7.324219\cdot10^{-6}\right)\ \boldsymbol{s}^2$$

Aby zmniejszyć wpływ bezwładności obrotowej
można przyjąć Ma ze współczynnikiem 0<a<1

$$a:=0.1\qquad Ma:=\begin{bmatrix}\frac{12}{\lambda e} & 0\\ 0 & a\cdot\lambda e\end{bmatrix}$$

Agregacja diagonalnej bezwymiarowej macierzy bezwładności

$$M:=Agrg\_B\left(n,Ma,M0,Ma\right)$$

$$M=\begin{array}{c|cccccccccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & \cdots & 17\\ \hline
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
1 & 0 & 0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
2 & 0 & 0 & 19.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
3 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
4 & 0 & 0 & 0 & 0 & 19.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
5 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 19.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & & \\
8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.2 & 0 & 0 & 0 & 0 & 0 & & \\
9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & & \\
10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.2 & 0 & 0 & 0 & & \\
11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & & \\
12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.2 & 0 & & \\
13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & & \\
\vdots & & & & & & & & & & & & & & & \ddots & \\
17 & & & & & & & & & & & & & & & &
\end{array}$$

Uwzględnienie warunków brzegowych

$$i:=1\,..\,Lr$$

$$w:=1\qquad K_{w,i}:=0\quad K_{i,w}:=0\quad K_{w,w}:=1\quad M_{w,i}:=0\quad M_{w,w}:=1$$

$$w:=Lr-1\qquad K_{w,i}:=0\quad K_{i,w}:=0\quad K_{w,w}:=1\quad M_{w,i}:=0\quad M_{w,w}:=1$$

$$\kappa\cdot\left[K-\omega^2\ \theta\cdot M\right]\ y=0\qquad \left|M^{-1}\cdot K-\sigma\cdot I\right|=0\qquad \omega^2\cdot\theta=\sigma\qquad |M|=9.172943\cdot10^2$$

$$\sigma:=\mathrm{eigenvals}\left(M^{-1}\cdot K\right)\qquad \omega:=\mathrm{sort}\left(\sqrt{\frac{\sigma}{\theta}}\right)$$

$$\omega^{\mathrm{T}}=\left[11.62\ \ 46.39\ \ 103.91\ \ 183.03\ \ 280.6\ \ 369.5\ \ 369.5\ \ 387.75\ \ 482.09\ \ 1652.47\ \ 1723.02\ \ 1900.84\ \ 2127.53\ \ 2358.89\ \ \ldots\right]\ \frac{\boldsymbol{rad}}{\boldsymbol{s}}$$

Równanie wymuszonych, tłumionych drgań belki

$$\psi_i = \frac{u_i}{l}$$

$$\begin{array}{c} e \\ i\ ||==============||\ j \\ \text{<--------- } Le \text{ ---------->} \end{array}$$

$$y_e = \begin{bmatrix} \psi_i \\ \varphi_i \\ \psi_j \\ \varphi_j \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \therefore \\ y_N \end{bmatrix}$$

$$K_@ \cdot u(t) + C_@ \cdot u'(t) + M_@ \cdot u''(t) = p(t) \qquad u' = \frac{\mathrm{d}}{\mathrm{d}t} u$$

$$C_@ = \alpha \cdot M_@ + \beta \cdot K_@$$ <--- *macierz tłumienia Rayleigha*

$$\begin{bmatrix} \frac{1}{\omega_1} & \omega_1 \\ \frac{1}{\omega_2} & \omega_2 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 2 \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$$ *Współczynniki (a,b) macierzy tłumienia*

$$C_@ = \alpha \cdot \eta \cdot M + \beta \cdot \kappa \cdot K$$

$$\kappa \cdot K \cdot y(t) + C_@ \cdot y'(t) + \eta \cdot M \cdot y''(t) = p(t)$$

$$K \cdot y(t) + \beta \cdot C \cdot y'(t) + \theta \cdot M \cdot y''(t) = f(t)$$

$$\omega_1 := \omega_{A_1} \qquad \omega_2 := \omega_{A_2} \qquad \zeta_1 := 0.05 \qquad \zeta_2 := 0.07$$

$$\kappa = 53.3\ \mathbf{kN} \qquad \eta = 0.391\ \mathbf{N} \cdot \mathbf{s}^2 \qquad \theta := \frac{\eta}{\kappa} = \left(7.324219 \cdot 10^{-6}\right)\ \mathbf{s}^2$$

$$\beta := 2\ \frac{\omega_1 \cdot \zeta_1 - \omega_2 \cdot \zeta_2}{\omega_1^{\ 2} - \omega_2^{\ 2}} \qquad \beta = \left(2.636532 \cdot 10^{-3}\right)\ \mathbf{s}$$

Macierze K, C, M, y, f - są bezwymiarowe

$$C := K + \frac{\alpha \cdot \theta}{\beta} \cdot M \qquad f(t) := \frac{1}{\kappa} \cdot p1(t)$$

$$\alpha := \omega_1 \cdot \left(2 \cdot \zeta_1 - \omega_1 \cdot \beta\right) \qquad \alpha = \left(8.064465 \cdot 10^{-1}\right)\ \frac{1}{\mathbf{s}}$$

$$C = \begin{bmatrix}
1.002 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 5 & -6 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & -9.6 & 6 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & -6 & 2.5 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9.6 & -6 & 19.243 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 2.5 & 0 & 10.001 & \\
 & & & & & & & & & & & & & & \ddots
\end{bmatrix}$$

Różnice centralne. *Chwile* (t-Δt), t, (t+Δt) *oznaczane są indeksami górnymi* -----> $y'_t = \dfrac{y^{t+\Delta t} - y^{t-\Delta t}}{2\ \Delta t}$ $\quad y''_t = \dfrac{y^{t+\Delta t} - 2\ y^t + y^{t-\Delta t}}{\Delta t^2}$

Po podstawieniu różnic zamiast pochodnych otrzymamy:

$t_{kr} := \dfrac{2}{\omega_{Lr-1}} = 0.000707\ s \qquad \Delta t := 0.0007\ s \qquad \Delta t < t_{kr} \qquad \dfrac{\Delta t}{t_{kr}} = 0.99$

$$K \cdot y^t + \frac{\beta}{2\ \Delta t} \cdot C \cdot \left(y^{t+\Delta t} - y^{t-\Delta t}\right) + \frac{\theta}{(\Delta t)^2} \cdot M \cdot \left(y^{t+\Delta t} - 2\ y^t + y^{t-\Delta t}\right) = f^t$$

$a_1 := \dfrac{(\Delta t)^2}{\theta} = 6.690133 \cdot 10^{-2} \qquad a_2 := \dfrac{\beta \cdot \Delta t}{2 \cdot \theta} = 1.259911 \cdot 10^{-1}$

$$a_1 \cdot K \cdot y^t + a_2 \cdot C \cdot \left(y^{t+\Delta t} - y^{t-\Delta t}\right) + M \cdot \left(y^{t+\Delta t} - 2\ y^t + y^{t-\Delta t}\right) = a_1 \cdot f^t$$

To równanie można względem czasu scałkować metodą "explicit":

$$y^{t+\Delta t} \cdot \left(M + a_2 \cdot C\right) = a_1 \cdot f^t - a_1 \cdot K \cdot y^t + a_2 \cdot C \cdot y^{t-\Delta t} + M \cdot \left(2 \cdot y^t - y^{t-\Delta t}\right)$$

$MC := M + a_2 \cdot C \qquad |MC| = 13029365854.651 \qquad Mc1 := MC^{-1}$

$$y^{t+\Delta t} = Mc1 \cdot \left[a_1 \cdot f^t - a_1 \cdot K \cdot y^t + a_2 \cdot C \cdot y^{t-\Delta t} + M \cdot \left(2 \cdot y^t - y^{t-\Delta t}\right)\right]$$

$M_K := Mc1 \cdot \left(2 \cdot M - a_1 \cdot K\right) \qquad M_C := Mc1 \cdot \left(a_2 \cdot C - M\right)$

$y^{t+\Delta t} = r_f{}^t + M_K \cdot y^t + M_C \cdot y^{t-\Delta t} \qquad r(t) := Mc1 \cdot a_1 \cdot f(t)$

Uwzględnienie warunków brzegowych

$i := 1..Lr$

$w := 1 \qquad M_{K_{w,i}} := 0 \qquad M_{K_{w,w}} := 1 \qquad M_{C_{w,i}} := 0 \qquad M_{C_{w,w}} := 1$

$w := Lr - 1 \qquad M_{K_{w,i}} := 0 \qquad M_{K_{w,w}} := 1 \qquad M_{C_{w,i}} := 0 \qquad M_{C_{w,w}} := 1$

$\text{Round}\left(\dfrac{11 \cdot \pi}{\omega_1 \cdot \Delta t}, 1\right) = 4247$

$Nst := 4000$ *<---- liczba kroków czasowych* $\qquad \Delta t = 0.0007\ s$

$Y_{Lr,2} := 0$ *<---- inicjacja macierzy rozwiazania i warunków brzegowych*

$$Explicit\_Central\_Time(A, N, \Delta) := \left\| \begin{array}{l} \text{for } i \in 1..N \\ \quad \left\| \begin{array}{l} k \leftarrow \text{cols}(A) \\ t \leftarrow (k-1) \cdot \Delta \\ y2 \leftarrow r(t) + M_K \cdot A^{\langle k \rangle} + M_C \cdot A^{\langle k-1 \rangle} \\ A \leftarrow \text{augment}(A, y2) \end{array} \right. \\ A \end{array} \right.$$

$y^{t+\Delta t} = r_f{}^t + M_K \cdot y^t + M_C \cdot y^{t-\Delta t}$

$Y := Explicit\_Central\_Time(Y, Nst, \Delta t)$ $\qquad i := 1..Nst$ $\qquad t_i := (i-1) \cdot \Delta t$

$t^T =$

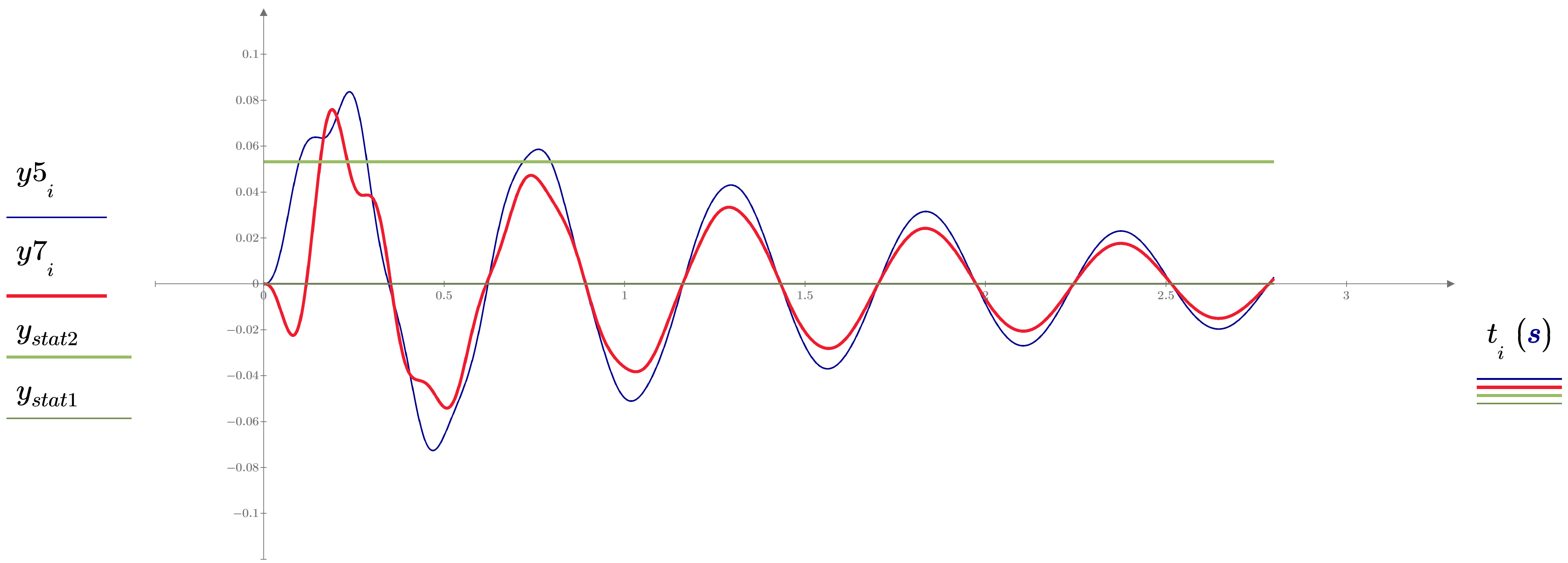
0 …	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	… 3999
…	1.456	1.4567	1.4574	1.4581	1.4588	1.4595	1.4602	1.4609	1.4616	1.4623	1.463	1.4637	1.4644	1.4651	1.4658	1.4665	1.4672	1.4679	…

s

$Y =$

	0 …	1768	1769	1770	1771	1772	1773	1774	1775	1776	1777	1778	1779	1780	1781	1782	1783	1784	1785	1786	… 4001
0	…	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1		0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	
2		0.014	0.014	0.014	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.016	
3		0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.012	0.012	
4		0.026	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.029	0.029	0.029	
5		0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	
6		0.035	0.035	0.035	0.035	0.035	0.035	0.036	0.036	0.036	0.036	0.036	0.037	0.037	0.037	0.037	0.037	0.037	0.038	0.038	
7		0.004	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	
8		0.037	0.038	0.038	0.038	0.038	0.038	0.039	0.039	0.039	0.039	0.04	0.04	0.04	0.04	0.04	0.041	0.041	0.041	0.041	
9		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
10		0.034	0.035	0.035	0.035	0.035	0.036	0.036	0.036	0.036	0.036	0.037	0.037	0.037	0.037	0.037	0.038	0.038	0.038	0.038	
11		−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	
12		0.026	0.027	0.027	0.027	0.027	0.027	0.027	0.028	0.028	0.028	0.028	0.028	0.028	0.029	0.029	0.029	0.029	0.029	0.029	
13		−0.008	−0.008	−0.008	−0.008	−0.008	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	−0.009	
⋮ 17																					⋱

$y5_i := Y_{7,i}$ $\qquad y7_i := Y_{13,i}$ $\qquad y_{stat2} := u_{max} = 0.053125$ $\qquad y_{stat1} := 0$

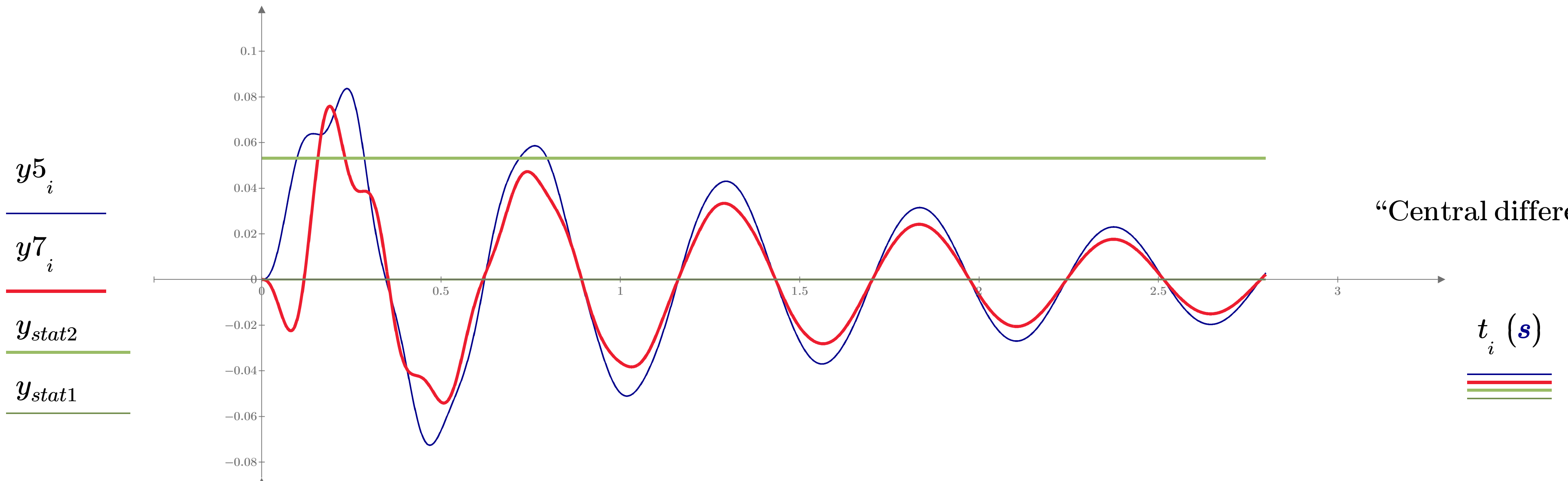


$\max(y5) = 0.083659$ $\qquad y_{inf} := y7_{Nst} = 0.002226$

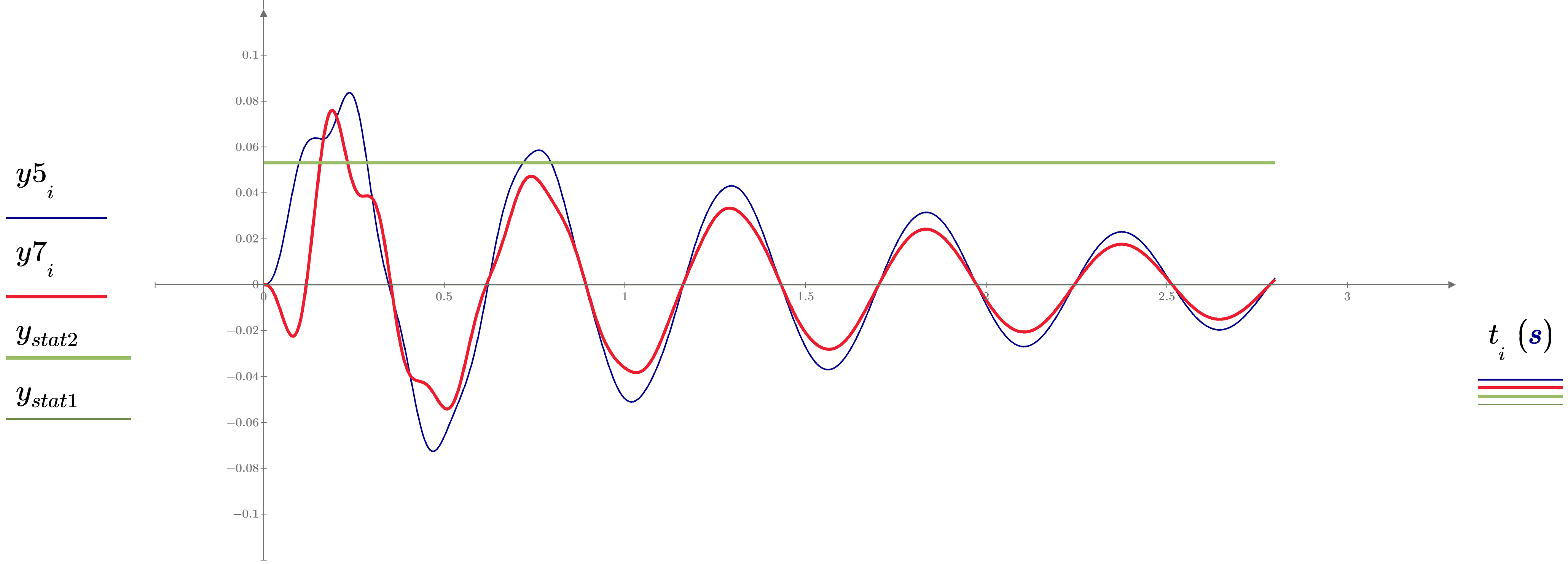
$$\frac{y_{inf}}{y_{stat2}} = 0.041893$$

$$\eta_{dyn} := \frac{\max(y7)}{y_{stat2}} = 1.429276$$

$$\eta_{dyn2} := 1.715522$$



“Central difference”



“Newmark”