

## Grupa 1

Wyznaczyć składowe macierzy sztywności elementów ramy płaskiej.

Podać postacie bloków A, B i C macierzy sztywności w lokalnym układzie współrzędnych z dokładnością do 5 miejsc znaczących [kN, kN/m, kNm]

$$E := 21 \text{ GPa} \quad b := 11 \text{ cm} \quad h := 17 \text{ cm}$$

$$J := \frac{b \cdot h^3}{36} = 1501.194 \text{ cm}^4 \quad A := \frac{b \cdot h}{2} = 93.500 \text{ cm}^2 \quad EJ := E \cdot J \quad EA := E \cdot A$$

$$EJ = 315.251 \text{ kN} \cdot \text{m}^2 \quad EA = 196350.0 \text{ kN}$$

Układ bloków macierzy sztywności elementu

$$K = \begin{bmatrix} A & C \\ C^T & B \end{bmatrix}$$

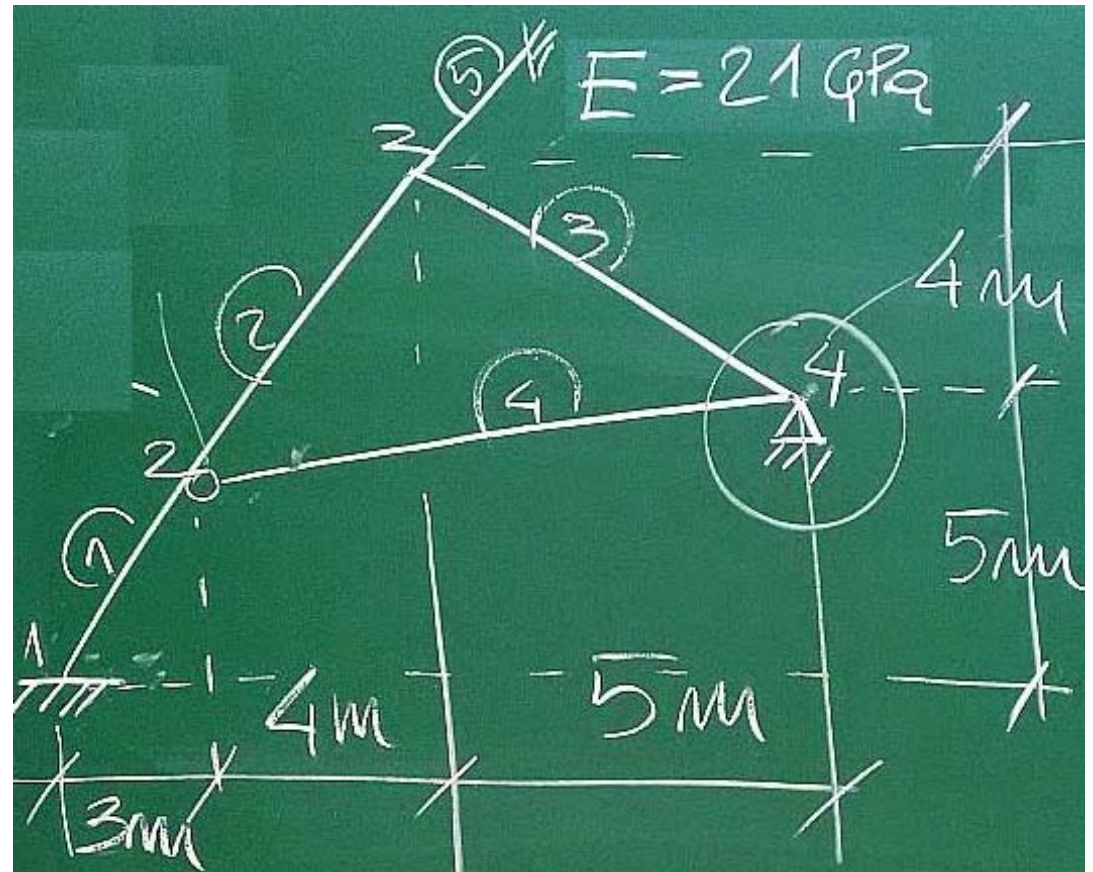
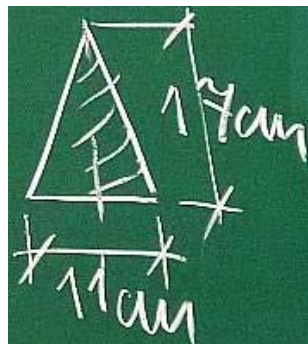
Warunki brzegowe (podporowe)

$$u_{X1} = 0 \quad u_{Y1} = 0 \quad \varphi_1 = 0$$

$$u_{X4} = 0 \quad u_{Y4} = 0$$

$$u_{X5} = 0 \quad u_{Y5} = 0 \quad \varphi_5 = 0$$

$$Y2 := 9 \text{ m} \cdot \frac{3}{7} = 3.85714 \text{ m}$$



## Element "1" - Bloki macierzy elementu bez przegubów

$$Lx := 3 \text{ m} = 3.00000 \text{ m} \quad Ly := Y2 = 3.85714 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 4.88647 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$A = \begin{bmatrix} 4.0182 \cdot 10^4 & 0 & 0 \\ 0 & 3.2423 \cdot 10 & (7.9217 \cdot 10) \text{ m} \\ 0 & (7.9217 \cdot 10) \text{ m} & (2.5806 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 4.0182 \cdot 10^4 & 0 & 0 \\ 0 & 3.2423 \cdot 10 & -7.9217 \cdot 10 \text{ m} \\ 0 & -7.9217 \cdot 10 \text{ m} & (2.5806 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -4.0182 \cdot 10^4 & 0 & 0 \\ 0 & -3.2423 \cdot 10 & (7.9217 \cdot 10) \text{ m} \\ 0 & -7.9217 \cdot 10 \text{ m} & (1.2903 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

Element "2" - Bloki macierzy elementu bez przegubów

$$Lx := 4 \text{ m} \quad Ly := 9 \text{ m} - Y2 = 5.14286 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 6.515288 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$A = \begin{bmatrix} 3.0137 \cdot 10^4 & 0 & 0 \\ 0 & 1.3678 \cdot 10 & (4.4559 \cdot 10) \text{ m} \\ 0 & (4.4559 \cdot 10) \text{ m} & (1.9355 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 3.0137 \cdot 10^4 & 0 & 0 \\ 0 & 1.3678 \cdot 10 & -4.4559 \cdot 10 \text{ m} \\ 0 & -4.4559 \cdot 10 \text{ m} & (1.9355 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -3.0137 \cdot 10^4 & 0 & 0 \\ 0 & -1.3678 \cdot 10 & (4.4559 \cdot 10) \text{ m} \\ 0 & -4.4559 \cdot 10 \text{ m} & (9.6773 \cdot 10) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

### Element "3" - Bloki macierzy elementu bez przegubów

$$Lx := 5 \text{ m} \quad Ly := -4 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 6.403124 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$A = \begin{bmatrix} 3.0665 \cdot 10^4 & 0 & 0 \\ 0 & 1.441 \cdot 10 & (4.6134 \cdot 10) \text{ m} \\ 0 & (4.6134 \cdot 10) \text{ m} & (1.9694 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 3.0665 \cdot 10^4 & 0 & 0 \\ 0 & 1.441 \cdot 10 & -4.6134 \cdot 10 \text{ m} \\ 0 & -4.6134 \cdot 10 \text{ m} & (1.9694 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -3.0665 \cdot 10^4 & 0 & 0 \\ 0 & -1.441 \cdot 10 & (4.6134 \cdot 10) \text{ m} \\ 0 & -4.6134 \cdot 10 \text{ m} & (9.8468 \cdot 10) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

Element "4" - Bloki macierzy elementu z przegubem w węźle początkowym

$$Lx := 9 \text{ m} \quad Ly := 5 \text{ m} - Y2 = 1.14286 \text{ m}$$

$$L := \sqrt{(Lx)^2 + (Ly)^2} = 9.072272 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2.1643 \cdot 10^4 & 0 & 0 \\ 0 & 1.2666 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & \frac{-3 EJ}{L^2} \\ 0 & \frac{-3 EJ}{L^2} & \frac{3 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 2.1643 \cdot 10^4 & 0 & 0 \\ 0 & 1.2666 & -1.1491 \cdot 10 \text{ m} \\ 0 & -1.1491 \cdot 10 \text{ m} & (1.0425 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-3 EJ}{L^3} & \frac{3 EJ}{L^2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -2.1643 \cdot 10^4 & 0 & 0 \\ 0 & -1.2666 & (1.1491 \cdot 10) \text{ m} \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$