

Grupa 2

Wyznaczyć składowe macierzy sztywności elementów ramy płaskiej.
Podać postacie bloków A, B i C macierzy sztywności w lokalnym układzie współrzędnych z dokładnością do 5 miejsc znaczących.

$$E := 23 \text{ GPa} \quad b := 13 \text{ cm} \quad h := 21 \text{ cm}$$

$$J := \frac{b \cdot h^3}{36} = 3344.250 \text{ cm}^4 \quad A := \frac{b \cdot h}{2} = 136.500 \text{ cm}^2 \quad EJ := E \cdot J \quad EA := E \cdot A$$

$$EJ = 769.178 \text{ kN} \cdot \text{m}^2 \quad EA = 313950.0 \text{ kN}$$

Układ bloków macierzy sztywności elementu

$$K = \begin{bmatrix} A & C \\ C^T & B \end{bmatrix}$$

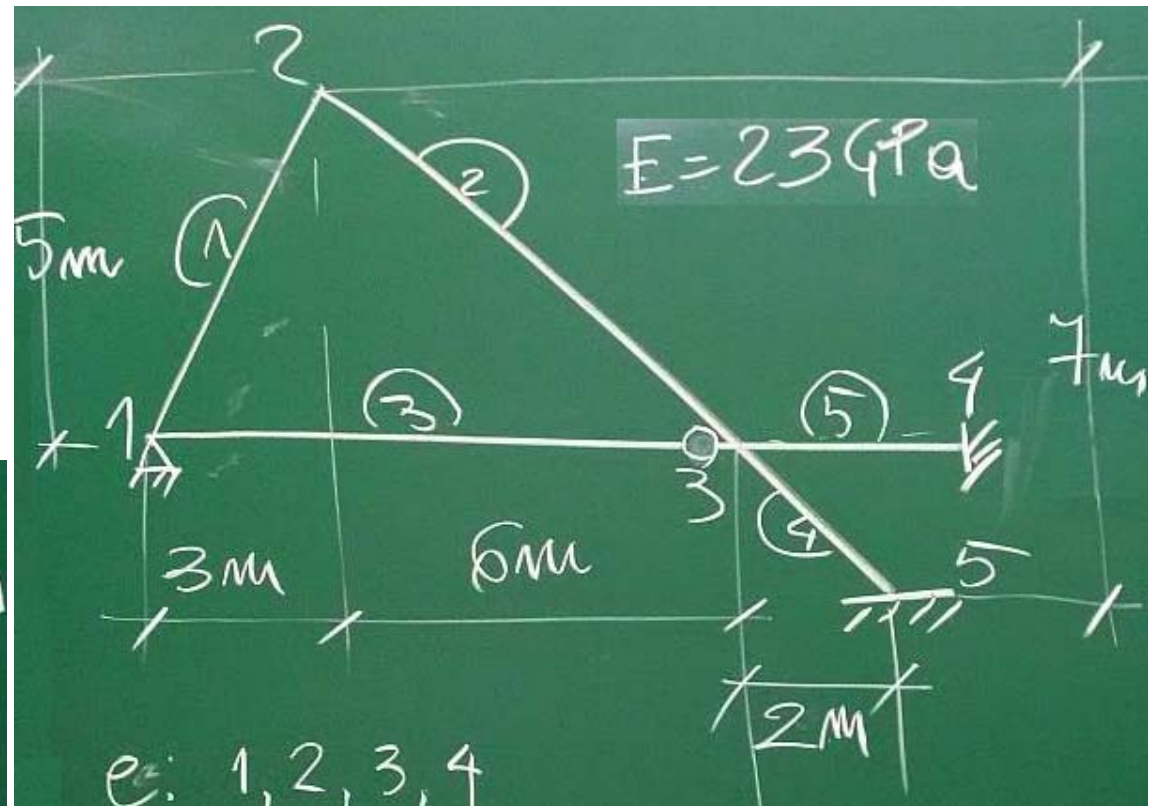
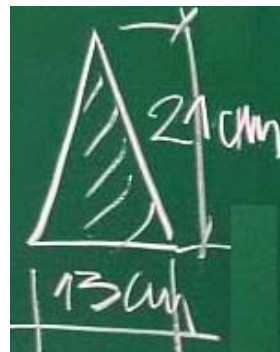
Warunki brzegowe (podporowe)

$$u_{X1} = 0 \quad u_{Y1} = 0$$

$$u_{X4} = 0 \quad u_{Y4} = 0 \quad \varphi_4 = 0$$

$$u_{X5} = 0 \quad u_{Y5} = 0 \quad \varphi_5 = 0$$

$$Y3 := 5 \text{ m} - 7 \text{ m} \cdot \frac{6}{8} = -0.25000 \text{ m}$$



Element "1" - Bloki macierzy elementu bez przegubów

$$Lx := 3 \text{ m} = 3.00000 \text{ m} \quad Ly := 5 \text{ m}$$

$$L := \sqrt{(Lx)^2 + (Ly)^2} = 5.83095 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$A = \begin{bmatrix} 5.3842 \cdot 10^4 & 0 & 0 \\ 0 & 4.6557 \cdot 10 & (1.3574 \cdot 10^2) \text{ m} \\ 0 & (1.3574 \cdot 10^2) \text{ m} & (5.2765 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 5.3842 \cdot 10^4 & 0 & 0 \\ 0 & 4.6557 \cdot 10 & -1.3574 \cdot 10^2 \text{ m} \\ 0 & -1.3574 \cdot 10^2 \text{ m} & (5.2765 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -5.3842 \cdot 10^4 & 0 & 0 \\ 0 & -4.6557 \cdot 10 & (1.3574 \cdot 10^2) \text{ m} \\ 0 & -1.3574 \cdot 10^2 \text{ m} & (2.6383 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

Element "2" - Bloki macierzy elementu bez przegubów

$$Lx := 6 \text{ m} \quad Ly := -7 \text{ m} \cdot \frac{6}{8} = -5.25000 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 7.972609 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$A = \begin{bmatrix} 3.9379 \cdot 10^4 & 0 & 0 \\ 0 & 1.8214 \cdot 10 & (7.2607 \cdot 10) \text{ m} \\ 0 & (7.2607 \cdot 10) \text{ m} & (3.8591 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 3.9379 \cdot 10^4 & 0 & 0 \\ 0 & 1.8214 \cdot 10 & -7.2607 \cdot 10 \text{ m} \\ 0 & -7.2607 \cdot 10 \text{ m} & (3.8591 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -3.9379 \cdot 10^4 & 0 & 0 \\ 0 & -1.8214 \cdot 10 & (7.2607 \cdot 10) \text{ m} \\ 0 & -7.2607 \cdot 10 \text{ m} & (1.9296 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

Element "4" - Bloki macierzy elementu bez przegubów

$$Lx := 2 \text{ m} \quad Ly := -7 \text{ m} \cdot \frac{2}{8} = -1.75000 \text{ m}$$

$$L := \sqrt{(Lx)^2 + (Ly)^2} = 2.657536 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$A = \begin{bmatrix} 1.1814 \cdot 10^5 & 0 & 0 \\ 0 & 4.9178 \cdot 10^2 & (6.5346 \cdot 10^2) \text{ m} \\ 0 & (6.5346 \cdot 10^2) \text{ m} & (1.1577 \cdot 10^3) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 1.1814 \cdot 10^5 & 0 & 0 \\ 0 & 4.9178 \cdot 10^2 & -6.5346 \cdot 10^2 \text{ m} \\ 0 & -6.5346 \cdot 10^2 \text{ m} & (1.1577 \cdot 10^3) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -1.1814 \cdot 10^5 & 0 & 0 \\ 0 & -4.9178 \cdot 10^2 & (6.5346 \cdot 10^2) \text{ m} \\ 0 & -6.5346 \cdot 10^2 \text{ m} & (5.7887 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

Element "3" - Bloki macierzy elementu z przegubem w węźle końcowym

$$Lx := 9 \text{ m}$$

$$Ly := Y3 = -0.25000 \text{ m}$$

$$L := \sqrt{(Lx)^2 + (Ly)^2} = 9.003472 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & \frac{3 EJ}{L^2} \\ 0 & \frac{3 EJ}{L^2} & \frac{3 EJ}{L} \end{bmatrix}$$

$$A = \begin{bmatrix} 3.487 \cdot 10^4 & 0 & 0 \\ 0 & 3.1617 & (2.8466 \cdot 10) \text{ m} \\ 0 & (2.8466 \cdot 10) \text{ m} & (2.5629 \cdot 10^2) \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 3.487 \cdot 10^4 & 0 & 0 \\ 0 & 3.1617 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-3 EJ}{L^3} & 0 \\ 0 & \frac{-3 EJ}{L^2} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -3.487 \cdot 10^4 & 0 & 0 \\ 0 & -3.1617 & 0 \\ 0 & -2.8466 \cdot 10 \text{ m} & 0 \end{bmatrix} \frac{kN}{m}$$