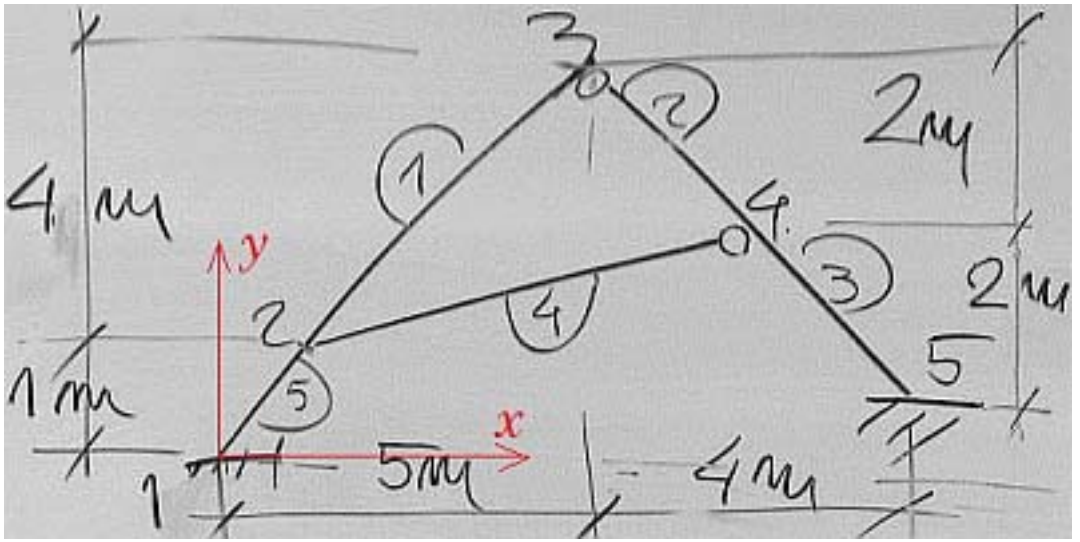
Wyznaczyć składowe macierzy sztywności elementów ramy płaskiej.
Podać postacie bloków A, B i C macierzy sztywności w lokalnym układzie współrzędnych z dokładnością do +/-0.5 [kN, kN/m, kNm]

Grupa A

$E := 12\ GPa$ $b := 7\ cm$ $h := 15\ cm$

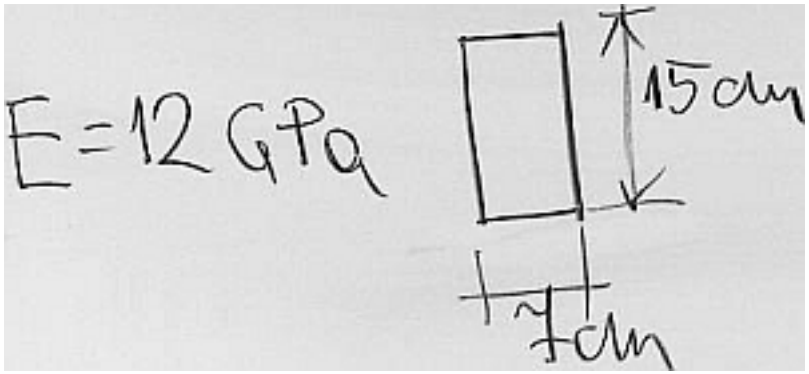
$J := \frac{b \cdot h^3}{12} = 1968.750\ cm^4$ $A := b \cdot h = 105.000\ cm^2$ $EJ := E \cdot J$ $EA := E \cdot A$

$EJ = 236.250\ kN \cdot m^2$ $EA = 126000.000\ kN$



Układ bloków macierzy sztywności elementu

$$K = \begin{bmatrix} A & C \\ C^{\mathrm{T}} & B \end{bmatrix}$$



Warunki brzegowe (podporowe)

$u_{X1} = 0$ $u_{Y1} = 0$ $\varphi_1 = 0$

$u_{X5} = 0$ $u_{Y5} = 0$ $\varphi_5 = 0$

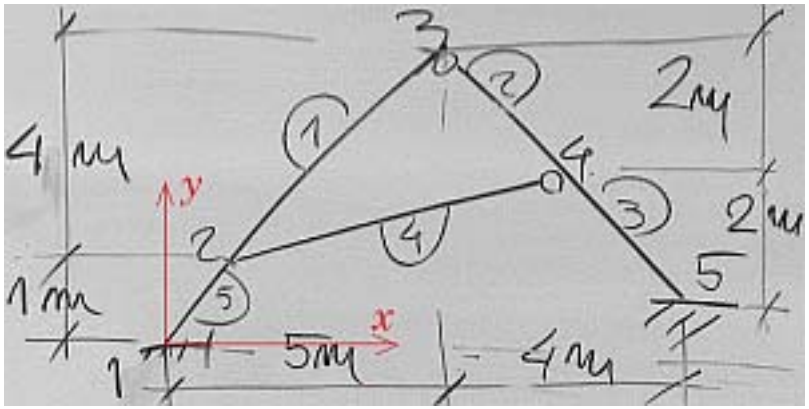
$X2 := 5\ m \cdot \frac{1}{5} = 1\ m$

$X4 := 9\ m - 2\ m = 7\ m$

Element "1" - Bloki macierzy elementu bez przegubów

$Lx := 5\ m - X2 = 4\ m \qquad Ly := 4\ m = 4\ m \qquad L := \sqrt{(Lx)^2 + (Ly)^2} = 5.656854\ m$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\ EJ}{L^3} & \frac{6\ EJ}{L^2} \\ 0 & \frac{6\ EJ}{L^2} & \frac{4\ EJ}{L} \end{bmatrix} \qquad A = \begin{bmatrix} 22274\ \frac{1}{m} & 0 & 0 \\ 0 & 16\ \frac{1}{m} & 44 \\ 0 & 44 & 167\ m \end{bmatrix} kN$$

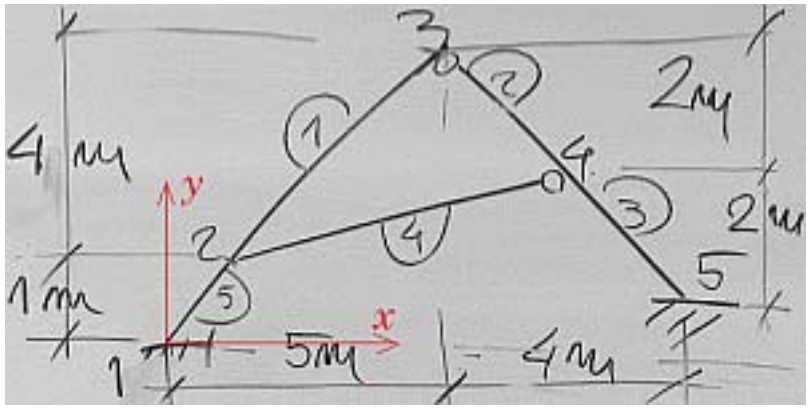


$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\ EJ}{L^3} & \frac{-6\ EJ}{L^2} \\ 0 & \frac{-6\ EJ}{L^2} & \frac{4\ EJ}{L} \end{bmatrix} \qquad B = \begin{bmatrix} 22274\ \frac{1}{m} & 0 & 0 \\ 0 & 16\ \frac{1}{m} & -44 \\ 0 & -44 & 167\ m \end{bmatrix} kN$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12\ EJ}{L^3} & \frac{6\ EJ}{L^2} \\ 0 & \frac{-6\ EJ}{L^2} & \frac{2\ EJ}{L} \end{bmatrix} \qquad C = \begin{bmatrix} -22274\ \frac{1}{m} & 0 & 0 \\ 0 & -16\ \frac{1}{m} & 44 \\ 0 & -44 & 84\ m \end{bmatrix} kN$$

Element "2" - Bloki macierzy elementu z przegubem w węźle początkowym

$Lx := 2\ m \qquad Ly := -2\ m \qquad L := \sqrt{(Lx)^2 + (Ly)^2} = 2.828427\ m$



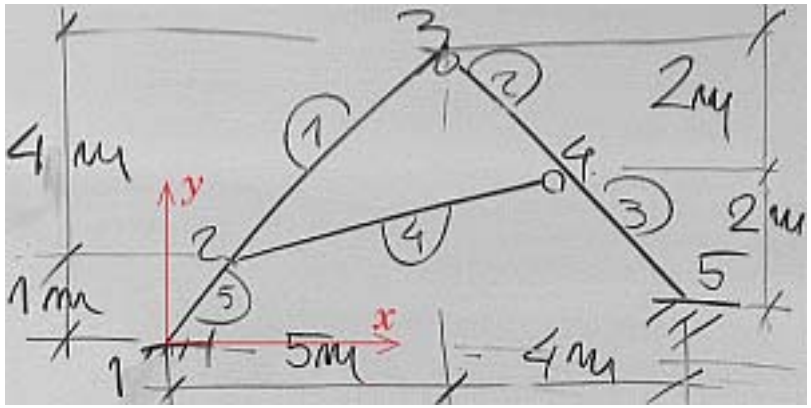
$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3\ EJ}{L^3} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} 44548 & 0 & 0 \\ 0 & 31 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3\ EJ}{L^3} & \frac{-3\ EJ}{L^2} \\ 0 & \frac{-3\ EJ}{L^2} & \frac{3\ EJ}{L} \end{bmatrix} \qquad B = \begin{bmatrix} 44548\ \frac{1}{m} & 0 & 0 \\ 0 & 31\ \frac{1}{m} & -89 \\ 0 & -89 & 251\ m \end{bmatrix} kN$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-3\ EJ}{L^3} & \frac{3\ EJ}{L^2} \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -44548\ \frac{1}{m} & 0 & 0 \\ 0 & -31\ \frac{1}{m} & 89 \\ 0 & 0 & 0 \end{bmatrix} kN$$

Element "3" - Bloki macierzy elementu bez przegubów

$Lx := 2\ m$ $\quad Ly := -2\ m$ $\quad L := \sqrt{(Lx)^2 + (Ly)^2} = 2.828427\ m$



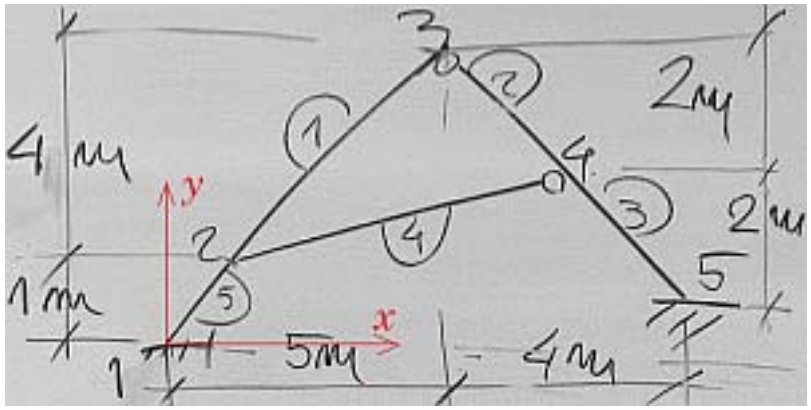
$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\ EJ}{L^3} & \frac{6\ EJ}{L^2} \\ 0 & \frac{6\ EJ}{L^2} & \frac{4\ EJ}{L} \end{bmatrix} \qquad A = \begin{bmatrix} 44548\ \frac{1}{m} & 0 & 0 \\ 0 & 125\ \frac{1}{m} & 177 \\ 0 & 177 & 334\ m \end{bmatrix} kN$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\ EJ}{L^3} & \frac{-6\ EJ}{L^2} \\ 0 & \frac{-6\ EJ}{L^2} & \frac{4\ EJ}{L} \end{bmatrix} \qquad B = \begin{bmatrix} 44548\ \frac{1}{m} & 0 & 0 \\ 0 & 125\ \frac{1}{m} & -177 \\ 0 & -177 & 334\ m \end{bmatrix} kN$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12\ EJ}{L^3} & \frac{6\ EJ}{L^2} \\ 0 & \frac{-6\ EJ}{L^2} & \frac{2\ EJ}{L} \end{bmatrix} \qquad C = \begin{bmatrix} -44548\ \frac{1}{m} & 0 & 0 \\ 0 & -125\ \frac{1}{m} & 177 \\ 0 & -177 & 167\ m \end{bmatrix} kN$$

Element "4" - Bloki macierzy elementu z przegubem w węźle końcowym

$Lx := X4 - X2 = 6\ m \qquad Ly := 2\ m \qquad L := \sqrt{(Lx)^2 + (Ly)^2} = 6.324555\ m$



$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3\ EJ}{L^3} & \frac{3\ EJ}{L^2} \\ 0 & \frac{3\ EJ}{L^2} & \frac{3\ EJ}{L} \end{bmatrix} \qquad A = \begin{bmatrix} 19922\ \frac{1}{m} & 0 & 0 \\ 0 & 3\ \frac{1}{m} & 18 \\ 0 & 18 & 112\ m \end{bmatrix} kN$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3\ EJ}{L^3} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 19922 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-3\ EJ}{L^3} & 0 \\ 0 & \frac{-3\ EJ}{L^2} & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -19922 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & -18\ m & 0 \end{bmatrix} \frac{kN}{m}$$