

Wyznaczyć składowe macierzy sztywności elementów ramy płaskiej.  
 Podać postacie bloków A, B i C macierzy sztywności w lokalnym układzie współrzędnych z dokładnością do +/-0.5 [kN, kN/m, kNm]

Grupa B

$$E := 12 \text{ GPa} \quad b := 9 \text{ cm} \quad h := 13 \text{ cm}$$

$$J := \frac{b \cdot h^3}{12} = 1647.750 \text{ cm}^4 \quad A := b \cdot h = 117.000 \text{ cm}^2 \quad EJ := E \cdot J \quad EA := E \cdot A$$

$$EJ = 197.730 \text{ kN} \cdot \text{m}^2 \quad EA = 140400.000 \text{ kN}$$

Układ bloków macierzy sztywności elementu

$$K = \begin{bmatrix} A & C \\ C^T & B \end{bmatrix}$$

Warunki brzegowe (podporowe)

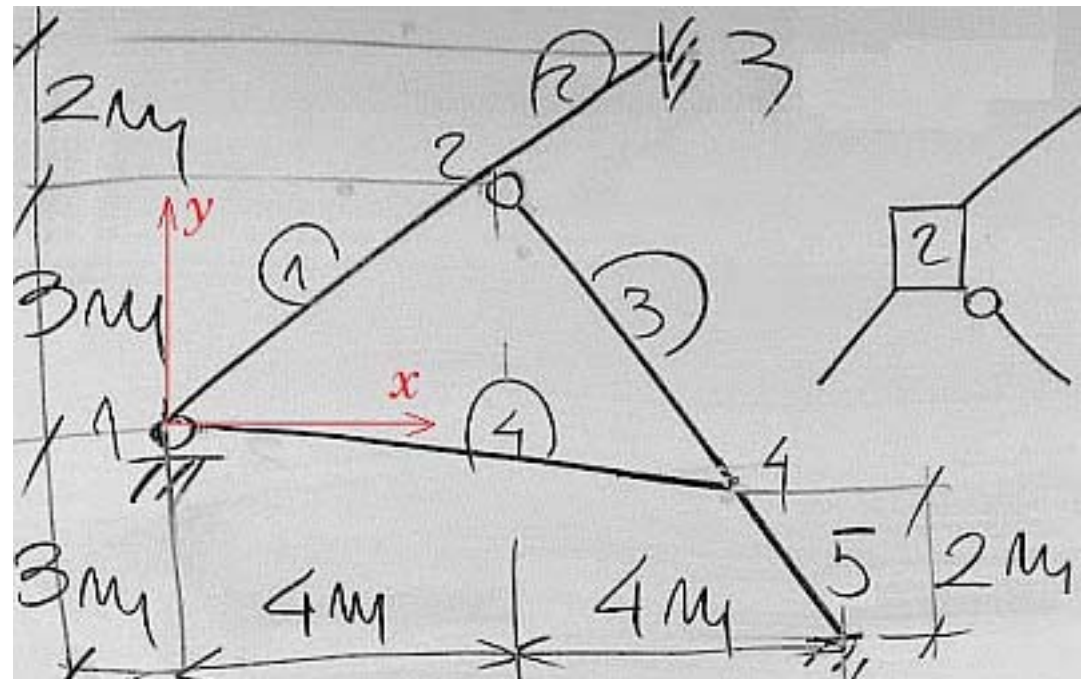
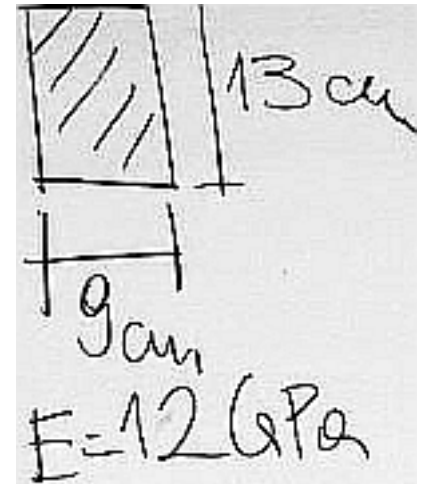
$$u_{X1} = 0 \quad u_{Y1} = 0$$

$$u_{X3} = 0 \quad u_{Y3} = 0 \quad \varphi_3 = 0$$

$$u_{X5} = 0 \quad u_{Y5} = 0 \quad \varphi_5 = 0$$

$$X3 := 4 \text{ m} \cdot \frac{5}{3} = 6.6667 \text{ m}$$

$$X4 := 8 \text{ m} - 4 \text{ m} \cdot \frac{2}{6} = 6.6667 \text{ m}$$



Element "1" - Bloki macierzy elementu bez przegubów

$$Lx := 4 \text{ m} = 4 \text{ m} \quad Ly := 3 \text{ m} = 3 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 5.000000 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

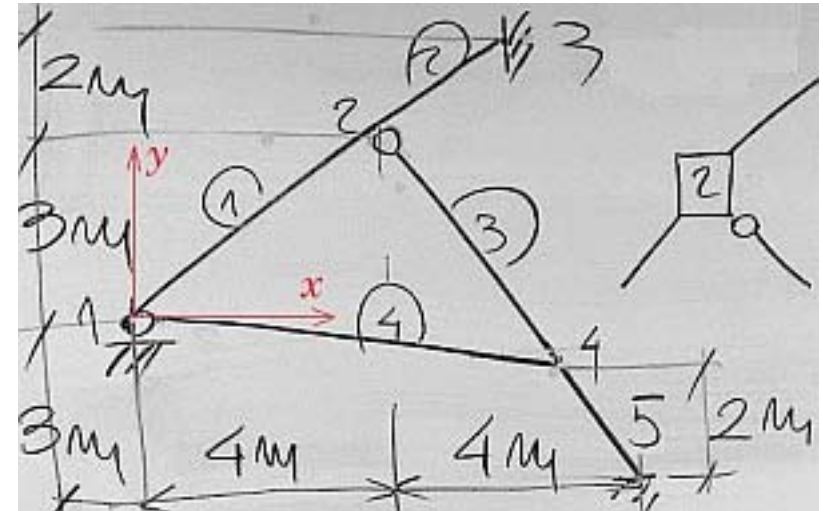
$$A = \begin{bmatrix} 28080 \frac{1}{m} & 0 & 0 \\ 0 & 19 \frac{1}{m} & 47 \\ 0 & 47 & 158 \text{ m} \end{bmatrix} \text{ kN}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 28080 \frac{1}{m} & 0 & 0 \\ 0 & 19 \frac{1}{m} & -47 \\ 0 & -47 & 158 \text{ m} \end{bmatrix} \text{ kN}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -28080 \frac{1}{m} & 0 & 0 \\ 0 & -19 \frac{1}{m} & 47 \\ 0 & -47 & 79 \text{ m} \end{bmatrix} \text{ kN}$$



## Element "2" - Bloki macierzy elementu bez przegubów

$$Lx := X3 - 4 \text{ m} = 2.667 \text{ m} \quad Ly := 2 \text{ m} = 2 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 3.333333 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

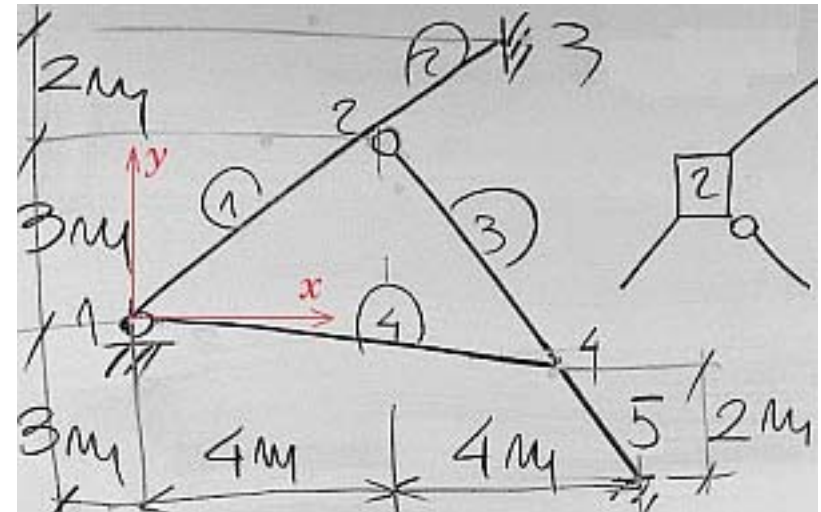
$$A = \begin{bmatrix} 42120 \frac{1}{m} & 0 & 0 \\ 0 & 64 \frac{1}{m} & 107 \\ 0 & 107 & 237 m \end{bmatrix} kN$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12 EJ}{L^3} & \frac{-6 EJ}{L^2} \\ 0 & \frac{-6 EJ}{L^2} & \frac{4 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 42120 \frac{1}{m} & 0 & 0 \\ 0 & 64 \frac{1}{m} & -107 \\ 0 & -107 & 237 m \end{bmatrix} kN$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-12 EJ}{L^3} & \frac{6 EJ}{L^2} \\ 0 & \frac{6 EJ}{L^2} & \frac{2 EJ}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} -42120 \frac{1}{m} & 0 & 0 \\ 0 & -64 \frac{1}{m} & 107 \\ 0 & -107 & 119 m \end{bmatrix} kN$$



Element "3" - Bloki macierzy elementu z przegubem w węźle początkowym

$$Lx := X4 - 4 \text{ m} = 2.667 \text{ m} \quad Ly := -4 \text{ m} = -4 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 4.807402 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

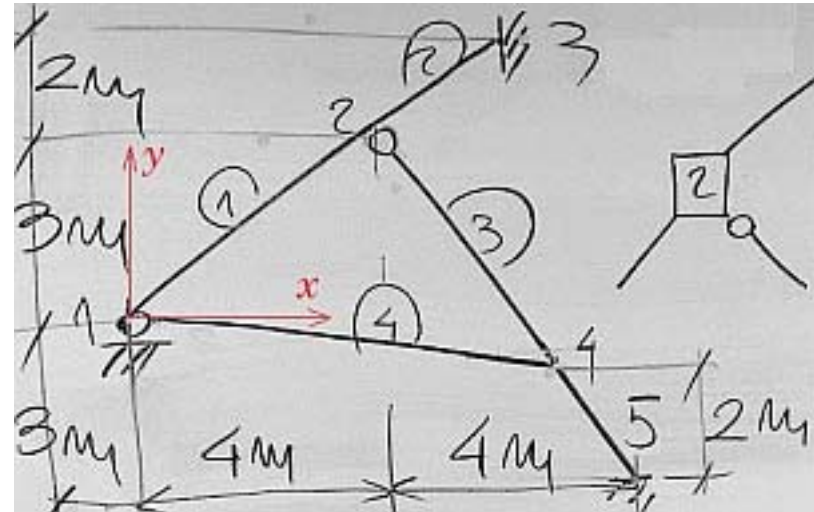
$$A = \begin{bmatrix} 29205 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & \frac{-3 EJ}{L^2} \\ 0 & \frac{-3 EJ}{L^2} & \frac{3 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 29205 & \frac{1}{m} & 0 & 0 \\ 0 & 5 & \frac{1}{m} & -26 \\ 0 & -26 & 123 & m \end{bmatrix} kN$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-3 EJ}{L^3} & \frac{3 EJ}{L^2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -29205 & 0 & 0 \\ 0 & -5 & 26 \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$



# Element "4" - Bloki macierzy elementu z przegubem w węźle początkowym

$$Lx := X4 = 6.667 \text{ m} \quad Ly := -1 \text{ m} = -1 \text{ m} \quad L := \sqrt{(Lx)^2 + (Ly)^2} = 6.741249 \text{ m}$$

$$A := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 20827 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$

$$B := \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{3 EJ}{L^3} & \frac{-3 EJ}{L^2} \\ 0 & \frac{-3 EJ}{L^2} & \frac{3 EJ}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 20827 & 0 & 0 \\ 0 & 2 & -13 \text{ m} \\ 0 & -13 \text{ m} & 88 \text{ m}^2 \end{bmatrix} \frac{kN}{m}$$

$$C := \begin{bmatrix} \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{-3 EJ}{L^3} & \frac{3 EJ}{L^2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -20827 & 0 & 0 \\ 0 & -2 & 13 \text{ m} \\ 0 & 0 & 0 \end{bmatrix} \frac{kN}{m}$$

