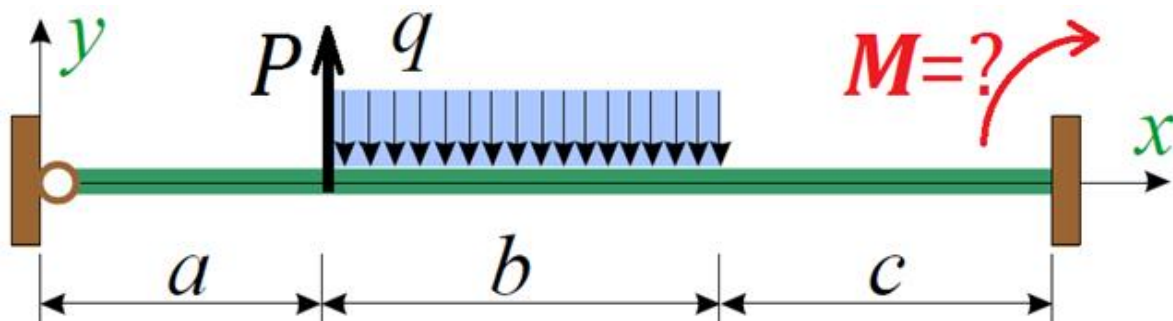


Należy obliczyć moment węzłowy  $M$  wykorzystując zasadę prac wirtualnych i funkcję kształtu elementu belkowego, dokładność  $\pm 0.005 \text{ kNm}$

$$q := 2.5 \cdot \frac{\text{kN}}{\text{m}} \quad P := 3 \text{ kN} \quad a := 2 \cdot \text{m} \quad b := 5 \cdot \text{m} \quad c := 2 \cdot \text{m} \quad L := a + b + c$$

$$\xi_1 := \frac{a}{L} = 0.222222 \quad \xi_2 := \frac{a+b}{L} = 0.777778 \quad \xi_P := \xi_1$$



$$K0001(\xi) := \frac{\xi}{2} \cdot (\xi^2 - 1)$$

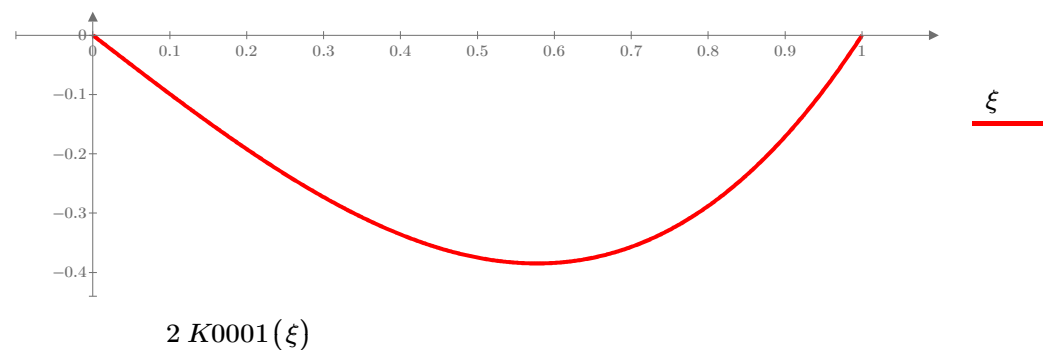
Równanie pracy wirtualnej

$$M_j \cdot 1 + PL \cdot K0001(\xi_P) - \int_{x_1}^{x_2} q \cdot L \cdot K0001(\xi) dx = 0$$

$$M_j \cdot 1 + PL \cdot K0001(\xi_P) - q \cdot L^2 \cdot \int_{\xi_1}^{\xi_2} K0001(\xi) d\xi = 0$$

$$M_j := q \cdot L^2 \cdot \int_{\xi_1}^{\xi_2} K0001(\xi) d\xi - P \cdot L \cdot K0001(\xi_P)$$

$$M := -M_j = 16.07 \text{ kN} \cdot \text{m}$$



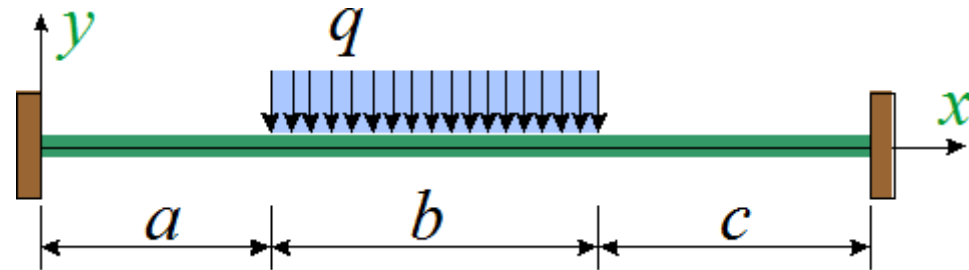
## Definicja wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej

$$H1000(\xi) := 1 - 3 \cdot \xi^2 + 2 \cdot \xi^3$$

$$H0100(\xi) := \xi \cdot (1 - 2 \cdot \xi + \xi^2)$$

$$H0010(\xi) := \xi^2 \cdot (3 - 2 \cdot \xi)$$

$$H0001(\xi) := -\xi^2 \cdot (1 - \xi)$$



Wyprowadzenie wzoru  $H1000(\xi)$

Warunki brzegowe:

$$y(0) = \delta \quad y'(0) = 0$$

$$y(l) = 0 \quad y'(l) = 0$$

$$y(x) = A + B \cdot x + C \cdot x^2 + D \cdot x^3$$

$$y'(x) = B + 2 C \cdot x + 3 D \cdot x^2$$

$$y(0) = A = \delta \quad y'(0) = B = 0$$

$$y(l) = \delta + C \cdot l^2 + D \cdot l^3 = 0$$

$$y'(l) = 2 C \cdot l + 3 D \cdot l^2 = 0$$

$$y(x) = \delta \cdot \left( 1 - 3 \cdot \frac{x^2}{l^2} + 2 \cdot \frac{x^3}{l^3} \right)$$

$$C \cdot l + D \cdot l^2 = -\frac{\delta}{l}$$

$$C \cdot l + \frac{3}{2} D \cdot l^2 = 0$$

$$\frac{1}{2} D \cdot l^2 = \frac{\delta}{l}$$

$$D = \frac{2 \delta}{l^3}$$

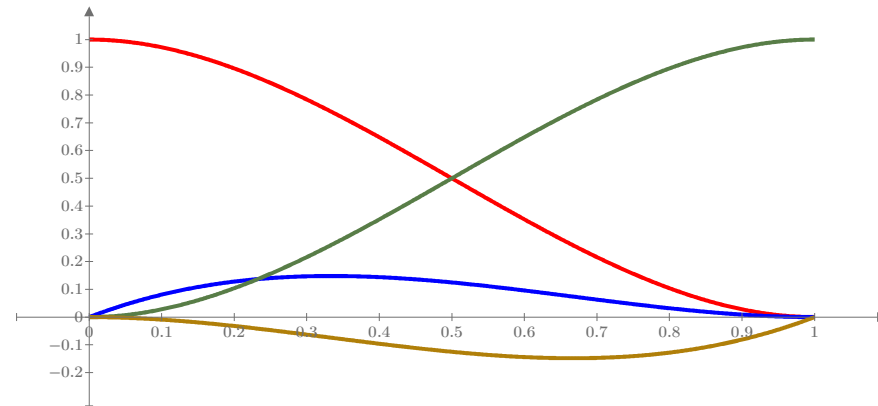
$$C = \frac{-3 \delta}{l^2}$$

$$y(\xi) = \delta \cdot (1 - 3 \cdot \xi^2 + 2 \cdot \xi^3)$$

$$y(\xi) = \delta \cdot H1000(\xi)$$

$$H1000(\xi) = (1 - 3 \cdot \xi^2 + 2 \cdot \xi^3)$$

$$\begin{array}{l} H1000(\xi) \\ H0100(\xi) \\ H0010(\xi) \\ H0001(\xi) \end{array}$$



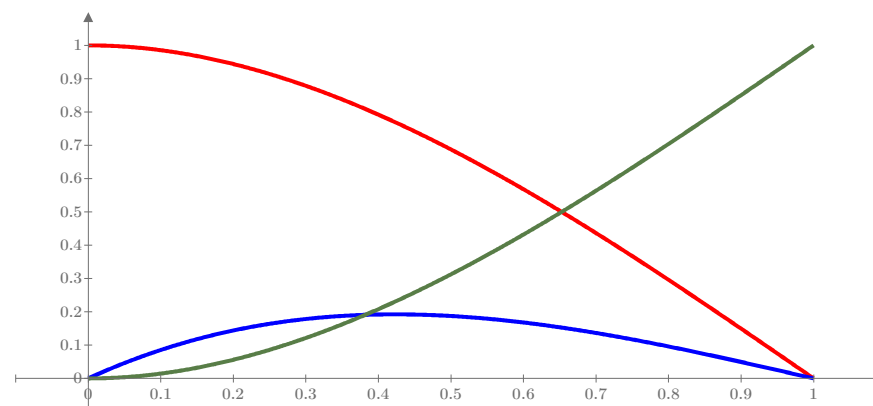
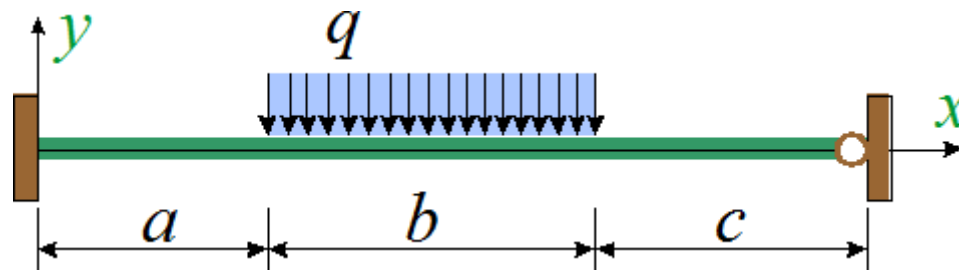
$\xi$

## Definicja wielomianów Hermite'a dla belki połączonej przegubowo z prawym węzłem - $G(\xi)$

$$G_{1000}(\xi) := 1 - \frac{3}{2} \cdot \xi^2 + \frac{1}{2} \cdot \xi^3$$

$$G_{0100}(\xi) := \frac{\xi}{2} \cdot (2 - 3 \cdot \xi + \xi^2)$$

$$G_{0010}(\xi) := \frac{\xi^2}{2} \cdot (3 - \xi)$$



$G_{1000}(\xi)$

$G_{0100}(\xi)$

$G_{0010}(\xi)$

$\xi$

$$y(0) = 0 \quad y'(0) = \varphi$$

$$y(l) = 0 \quad y''(l) = 0$$

$$y(x) = A + B \cdot x + C \cdot x^2 + D \cdot x^3$$

$$y'(x) = B + 2 C \cdot x + 3 D \cdot x^2$$

$$y''(x) = 2 C + 6 D \cdot x$$

$$y(0) = A = 0 \quad y'(0) = B = \varphi$$

$$y(l) = \varphi \cdot l + C \cdot l^2 + D \cdot l^3 = 0$$

$$y''(l) = 2 C + 6 D \cdot l = 0$$

$$\left. \begin{array}{l} C + D \cdot l = -\frac{\varphi}{l} \\ C + 3 D \cdot l = 0 \end{array} \right\} \rightarrow 2 D \cdot l = \frac{\varphi}{l^2} \rightarrow D = \frac{\varphi}{2 l^2} \rightarrow C = \frac{-3 \varphi}{2 l}$$

$$y(x) = \varphi \cdot \left( x - \frac{3}{2} \cdot \frac{x^2}{l} + \frac{x^3}{2 l^2} \right)$$

$$\rightarrow y(\xi) = \frac{\varphi \cdot l}{2} (2 \xi - 3 \cdot \xi^2 + \xi^3)$$

$$\rightarrow y(\xi) = \varphi \cdot l G_{0100}(\xi)$$

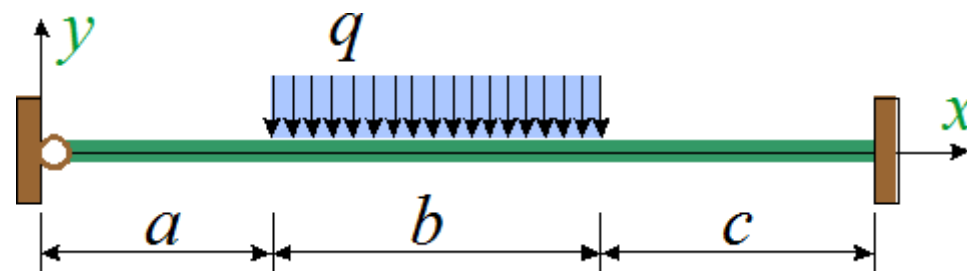
$$\rightarrow G_{0100}(\xi) = \frac{\xi}{2} \cdot (2 - 3 \cdot \xi + \xi^2)$$

*Definicja wielomianów Hermite'a dla belki połączonej przegubowo z lewym węzłem -  $K(\xi)$*

$$K1000(\xi) := 1 - \frac{3}{2} \cdot \xi + \frac{1}{2} \cdot \xi^3$$

$$K0010(\xi) := \frac{\xi}{2} \cdot (3 - \xi^2)$$

$$K0001(\xi) := \frac{\xi}{2} \cdot (\xi^2 - 1)$$



*Wyprowadzenie wzoru  $K1000(\xi)$*

*Warunki brzegowe:*

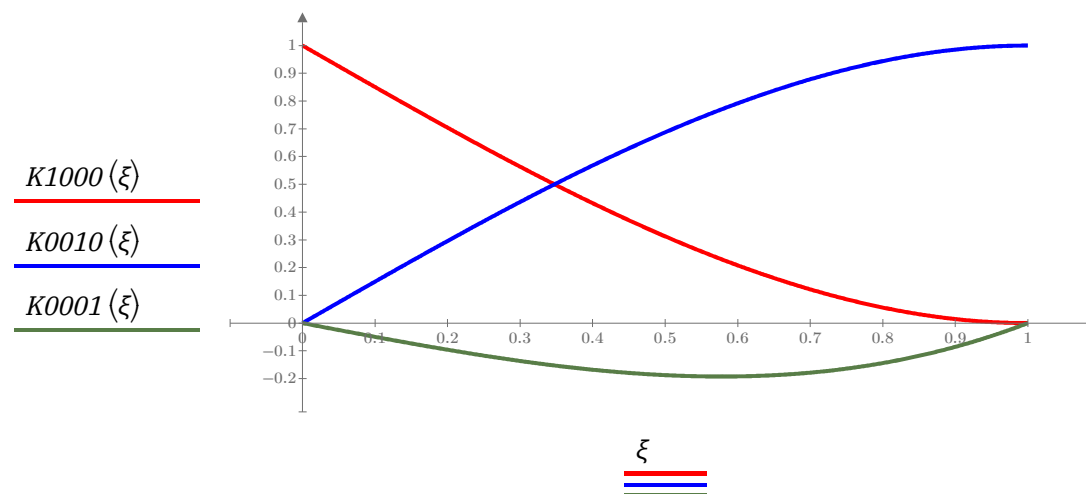
$$y(0) = \delta \qquad y''(0) = 0$$

$$y(l) = 0 \qquad y'(l) = 0$$

$$y(x) = A + B \cdot x + C \cdot x^2 + D \cdot x^3$$

$$y'(x) = B + 2 C \cdot x + 3 D \cdot x^2$$

$$y''(x) = 2C + 6D \cdot x$$



$$y(0) = A = \delta \quad y''(0) = 2 \quad C = 0$$

$$y(l) = \delta + B \cdot l + D \cdot l^3 = 0$$

$$y'(l) = B + 3 D \cdot l^2 = 0$$

$$\left. \begin{array}{l} \text{---} \rightarrow B + D \cdot l^2 = -\frac{\delta}{l} \\ \text{---} \rightarrow B + 3 D \cdot l^2 = 0 \end{array} \right\} \text{---} \rightarrow 2 D \cdot l^2 = \frac{\delta}{l} \text{---} \rightarrow D = \frac{\delta}{2 l^3} \text{---} \rightarrow B = \frac{-3 \delta}{2 l}$$

$$y(x) = \delta \cdot \left( 1 - \frac{3}{2} \cdot \frac{x}{l} + \frac{x^3}{2 l^3} \right)$$

----->  $y(\xi) = \delta \left( 1 - \frac{3}{2} \cdot \xi + \frac{1}{2} \xi^3 \right)$

----->  $y(\xi) = \delta K1000(\xi)$

$$\text{---} \rightarrow K_{1000}(\xi) = 1 - \frac{3}{2} \cdot \xi + \frac{1}{2} \xi^3$$