Example 5.E1.

The content of the example

A 3D frame with the scheme presented in Fig.5.E1.1 is loaded with the following static loads:

- concentrated force equal to 60 kN applied to bar No 1;
- uniform distributed load with the value of 15 kN/m applied to bar No 2;
- concentrated moment equal to 20 kNm applied to the bar No 3.

The shape of the cross sections of frame elements is shown in Fig.5.E1.1. Geometric characteristics of the cross section and material constants are given in Tab.5.E1.1. Nodal coordinates are given in Tab.5.E1.2 whereas Tab.5.E1.3 gives global numbers of nodes n_i and n_j of elements, lengths of their projections (L_X, L_Y, L_Z) on the axes X, Y and Z of the global coordinate system and direction cosines (c_X, c_Y, c_Z).

Determine nodal displacements of the frame and reactions of constraint supports and draw graphs of internal forces.

Tab.5.E1.1

Material constants	$E=2\cdot10^8$ kPa	$G=7.7\cdot10^7$ kPa	$\nu=0.3$	
Geometric characteristics	$A=0.036$ m^2	$J_s=12935\cdot10^{-8}$ m^4	$J_y=27000\cdot10^{-8}$ m^4	$J_z=4320\cdot10^{-8}$ m^4

Tab.5.E1.2

Node No n	X_n [m]	Y_n [m]	Z_n [m]
1	0.0	0.0	3.0
2	3.0	0.0	3.0
3	3.0	3.0	3.0
4	1.5	3.0	0.0

Tab.5.E1.3

Elem. No n	Node No ni	nj	L_{nX} [m]	L_{nY} [m]	L_{nZ} [m]	L_n [m]	c_X	c_Y	c_Z
1	1	2	3.0	0.0	0.0	3.0	1.0	0.0	0.0
2	2	3	0.0	3.0	0.0	3.0	0.0	1.0	0.0
3	4	3	1.5	0.0	3.0	3.3541	0.44721	0.0	0.89443

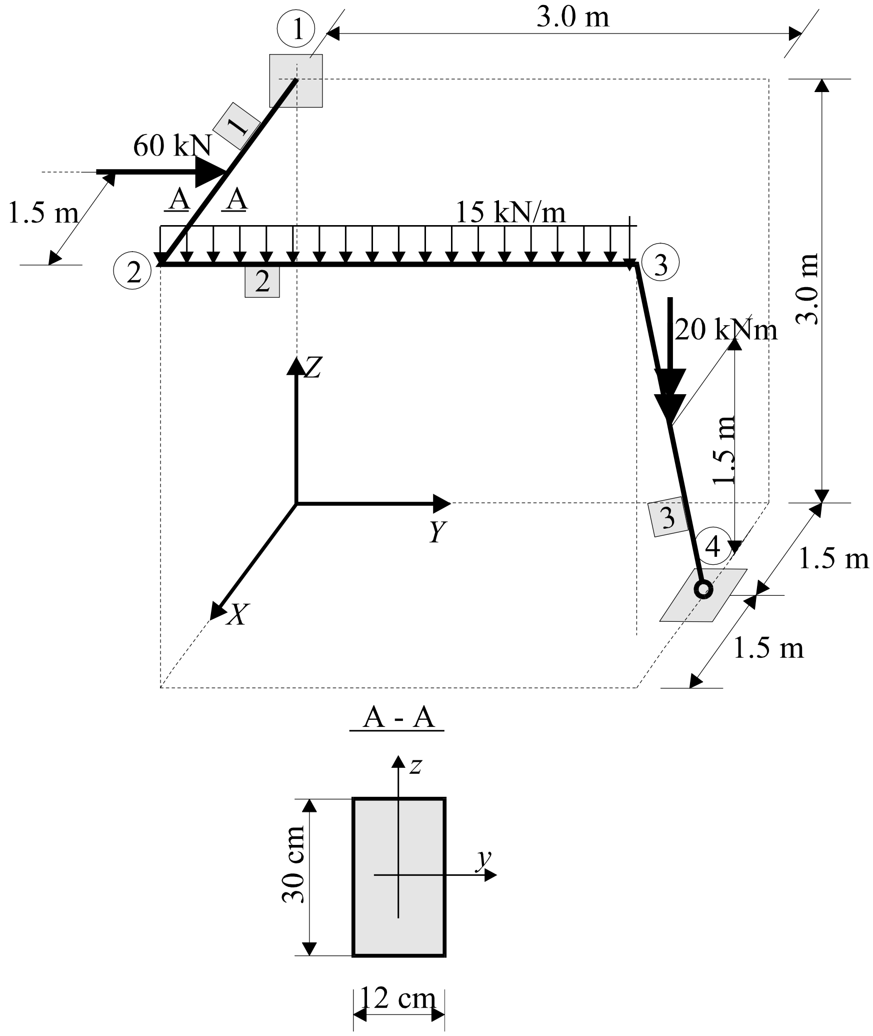

Fig.5.E1.1

The solution of the example

As in previous examples we start solving the problem from determining vectors of nodal forces caused by external loads. The values of components of these vectors in the global coordinate system *XYZ* are following:

$$
\mathbf{f}^1 = \left[\begin{array}{c} 0.0 \\ -30.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ -22.5 \\ \hline 0.0 \\ -30.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 22.5 \end{array}\right]\begin{array}{c} \\ \\ 1 \\ \\ \\ \\ \\ \\ 2 \\ \\ \\ \\ \end{array}
\qquad
\mathbf{f}^2 = \left[\begin{array}{c} 0.0 \\ 0.0 \\ 22.5 \\ 11.25 \\ 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ 22.5 \\ -11.25 \\ 0.0 \\ 0.0 \end{array}\right]\begin{array}{c} \\ \\ 2 \\ \\ \\ \\ \\ \\ 3 \\ \\ \\ \\ \end{array}
\qquad
\mathbf{f}^3 = \left[\begin{array}{c} 0.0 \\ -4.0049 \\ 0.0 \\ 6.0073 \\ 0.0 \\ 7.0086 \\ \hline 0.0 \\ 4.0049 \\ 0.0 \\ 6.0073 \\ 0.0 \\ 7.0086 \end{array}\right]\begin{array}{c} \\ \\ 4 \\ \\ \\ \\ \\ \\ 3 \\ \\ \\ \\ \end{array}
$$

The values of components are calculated by using equation (4.53) (comp. examples 1, 2 and 3 in Chapter IV) and next they are transformed to the global coordinate system applying the equation:

$$\mathbf{f}^e = \mathbf{R}^e \mathbf{f}'^e$$

where $\mathbf{R}^e$ is the rotation matrix described by equation (5.33) (comp. equation (5.24), too).

Using these vectors we can juxtapose the global vector of nodal forces for the frame:

$$
\mathbf{p}=\left[\begin{array}{c}0.0\\0.0\\0.0\\0.0\\0.0\\0.0\\\hline 0.0\\0.0\\0.0\\0.0\\0.0\\0.0\\\hline 0.0\\0.0\\0.0\\0.0\\0.0\\0.0\\\hline 0.0\\0.0\\0.0\\0.0\\0.0\\0.0\end{array}\right]
-\left[\begin{array}{c}0.0\\-30.0\\0.0\\0.0\\0.0\\-22.5\\\hline 0.0\\-30.0\\0.0\\0.0\\0.0\\22.5\\\hline \\ \\ \\ \\ \\ \\\hline \\ \\ \\ \\ \\ \end{array}\right]
-\left[\begin{array}{c} \\ \\ \\ \\ \\ \\\hline 0.0\\0.0\\22.5\\11.25\\0.0\\0.0\\\hline 0.0\\0.0\\22.5\\-11.25\\0.0\\0.0\\\hline \\ \\ \\ \\ \\ \end{array}\right]
-\left[\begin{array}{c} \\ \\ \\ \\ \\ \\\hline \\ \\ \\ \\ \\ \\\hline 0.0\\4.0049\\0.0\\6.0073\\0.0\\7.0086\\\hline 0.0\\-4.0049\\0.0\\6.0073\\0.0\\7.0086\end{array}\right]
=\left[\begin{array}{c}0.0\\30.0\\0.0\\0.0\\0.0\\22.5\\\hline 0.0\\30.0\\-22.5\\-11.25\\0.0\\-22.5\\\hline 0.0\\-4.0049\\-22.5\\5.2427\\0.0\\-7.0086\\\hline 0.0\\4.0049\\0.0\\-6.0073\\0.0\\-7.0086\end{array}\right]
\begin{array}{c} \\ \\ 1\\ \\ \\ \\ \\ \\ 2\\ \\ \\ \\ \\ \\ 3\\ \\ \\ \\ \\ \\ 4\\ \\ \\ \\ \end{array}
$$

After considering the boundary conditions

- at node No 1 $u_{1X}=0$ (1),

  $u_{1Y}=0$ (2),

  $u_{1Z}=0$ (3),

  $\varphi_{1X}=0$ (4),

  $\varphi_{1Y}=0$ (5),

$\varphi_{1Z}=0$ (6),

– at node No 4 $u_{4X}=0$ (19),

$u_{4Y}=0$ (20),

$u_{4Z}=0$ (21),

in which the global numbers of suitable degrees of freedom are given in brackets we obtain the vector:

$$\mathbf{p}^r = \left[\begin{array}{c} \mathit{0.0} \\ \mathit{0.0} \\ \mathit{0.0} \\ \mathit{0.0} \\ \mathit{0.0} \\ \mathit{0.0} \\ \hline 0.0 \\ 30.0 \\ -22.5 \\ -11.25 \\ 0.0 \\ -22.5 \\ \hline 0.0 \\ -4.0049 \\ -22.5 \\ 5.2427 \\ 0.0 \\ -7.0086 \\ \hline \mathit{0.0} \\ \mathit{0.0} \\ \mathit{0.0} \\ -6.0073 \\ 0.0 \\ -7.0086 \end{array}\right] \begin{array}{c} \\ \\ 1 \\ \\ \\ \\ \\ \\ 2 \\ \\ \\ \\ \\ \\ 3 \\ \\ \\ \\ \\ \\ 4 \\ \\ \\ \\ \end{array}$$

Modifications of the components of the vector $\mathbf{p}$ leading to the vector $\mathbf{p}^r$ are marked in italic fonts.

Determining components of the element stiffness matrices for the frame is the next step. Now we use equation (5.10) in order to find components in the local reference system *xyz* and to transform them to the global system *XYZ* (comp. equation (5.34)). After these operations we obtain matrices $\mathbf{K}^1$, $\mathbf{K}^2$, $\mathbf{K}^3$ presented on the successive pages.

The aggregation of the global stiffness matrix procceds according to the scheme:

$$
\mathbf{K}= \begin{array}{cc} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} & \\ \left[\begin{array}{c|c|c|c} \mathbf{K}^1_{11} & \mathbf{K}^1_{12} & & \\ \hline \mathbf{K}^1_{21} & \mathbf{K}^1_{22}+\mathbf{K}^2_{22} & \mathbf{K}^2_{23} & \\ \hline & \mathbf{K}^2_{32} & \mathbf{K}^2_{33}+\mathbf{K}^3_{33} & \mathbf{K}^3_{34} \\ \hline & & \mathbf{K}^3_{43} & \mathbf{K}^3_{44} \end{array}\right] & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array},
$$

where $\mathbf{K}^e_{ij}$ denotes the block of the stiffness matrix for the element e which is connected with forces at the node i induced by displacements of the node j. After inserting the values of components of the matrix, we obtain the matrix $\mathbf{K}$ (shown on the successive page after matrices $\mathbf{K}^e$) and after taking into consideration previously described boundary conditions, we obtain matrix $\mathbf{K}^r$ (presented after the matrix $\mathbf{K}$) which is used to solving the set of equations of the finite element method:

$$\mathbf{K}^r\,\mathbf{u} = \mathbf{p}^r .$$

The stiffness matrix of element No 1:

$$
\mathbf{K}^{1}=\begin{array}{c}
\begin{array}{cc} \qquad\qquad\qquad 1 \qquad\qquad\qquad\qquad\qquad\qquad & \qquad\qquad\qquad 2 \qquad\qquad\qquad \end{array} \\
\left[\begin{array}{cccccc|cccccc}
0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 3840.0 & 0.0 & 0.0 & 0.0 & 5760.0 & 0.0 & -3840.0 & 0.0 & 0.0 & 0.0 & 5760.0 \\
0.0 & 0.0 & 24000.0 & 0.0 & -36000.0 & 0.0 & 0.0 & 0.0 & -24000.0 & 0.0 & -36000.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 3316.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3316.7 & 0.0 & 0.0 \\
0.0 & 0.0 & -36000.0 & 0.0 & 72000.0 & 0.0 & 0.0 & 0.0 & 36000.0 & 0.0 & 36000.0 & 0.0 \\
0.0 & 5760.0 & 0.0 & 0.0 & 0.0 & 11520.0 & 0.0 & -5760.0 & 0.0 & 0.0 & 0.0 & 5760.0 \\
\hline
-0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -3840.0 & 0.0 & 0.0 & 0.0 & -5760.0 & 0.0 & 3840.0 & 0.0 & 0.0 & 0.0 & -5760.0 \\
0.0 & 0.0 & -24000.0 & 0.0 & 36000.0 & 0.0 & 0.0 & 0.0 & 24000.0 & 0.0 & 36000.0 & 0.0 \\
0.0 & 0.0 & 0.0 & -3316.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3316.7 & 0.0 & 0.0 \\
0.0 & 0.0 & -36000.0 & 0.0 & 36000.0 & 0.0 & 0.0 & 0.0 & 36000.0 & 0.0 & 72000.0 & 0.0 \\
0.0 & 5760.0 & 0.0 & 0.0 & 0.0 & 5760.0 & 0.0 & -5760.0 & 0.0 & 0.0 & 0.0 & 11520.0
\end{array}\right]
\end{array}
\begin{array}{c} 1 \\ \\ \\ \\ \\ \\ 2 \end{array}
$$

The stiffness matrix of element No 2:

$$\mathbf{K}^2=\begin{array}{cc} \begin{array}{cc} \hspace{12em} 2 \hspace{12em} & \hspace{12em} 3 \hspace{12em} \end{array} & \\ \left[\begin{array}{cccccc|cccccc} 3840.0 & 0.0 & 0.0 & 0.0 & 0.0 & -5760.0 & -3840.0 & 0.0 & 0.0 & 0.0 & 0.0 & -5760.0 \\ 0.0 & 0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 24000.0 & 36000.0 & 0.0 & 0.0 & 0.0 & 0.0 & -24000.0 & 36000.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 36000.0 & 72000.0 & 0.0 & 0.0 & 0.0 & 0.0 & -36000.0 & 36000.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 3316.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3316.7 & 0.0 \\ -5760.0 & 0.0 & 0.0 & 0.0 & 0.0 & 11520.0 & 5760.0 & 0.0 & 0.0 & 0.0 & 0.0 & 5760.0 \\ \hline -3840.0 & 0.0 & 0.0 & 0.0 & 0.0 & 5760.0 & 3840.0 & 0.0 & 0.0 & 0.0 & 0.0 & 5760.0 \\ 0.0 & -0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.24E7 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -24000.0 & -36000.0 & 0.0 & 0.0 & 0.0 & 0.0 & 24000.0 & -36000.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 36000.0 & 36000.0 & 0.0 & 0.0 & 0.0 & 0.0 & -36000.0 & 72000.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -3316.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3316.7 & 0.0 \\ -5760.0 & 0.0 & 0.0 & 0.0 & 0.0 & 5760.0 & 5760.0 & 0.0 & 0.0 & 0.0 & 0.0 & 11520.0 \end{array}\right] & \begin{array}{c} 2 \\ \\ \\ \\ \\ \\ 3 \end{array} \end{array}$$

The stiffness matrix of element No 3:

$$\mathbf{K}^3=\begin{array}{cc}
\begin{array}{cc}\qquad\qquad\qquad\qquad 4\qquad\qquad\qquad\qquad & \qquad\qquad\qquad\qquad 3\qquad\qquad\qquad\qquad\end{array} & \\
\left[\begin{array}{cccccc|cccccc}
4.4306E5 & 0.0 & 8.5178E5 & 0.0 & 25760. & 0.0 & -4.4306E5 & 0.0 & -8.5178E5 & 0.0 & 25760.0 & 0.0 \\
0.0 & 2747.7 & 0.0 & -4121.5 & 0.0 & 2060.8 & 0.0 & -2747.7 & 0.0 & -4121.5 & 0.0 & 2060.8 \\
8.5178E5 & 0.0 & 1.7207E6 & 0.0 & -12880.0 & 0.0 & -8.5178E5 & 0.0 & -1.7207E6 & 0.0 & -12880.0 & 0.0 \\
0.0 & -4121.5 & 0.0 & 8836.3 & 0.0 & -2934.9 & 0.0 & 4121.5 & 0.0 & 3528.2 & 0.0 & -3247.4 \\
25760.0 & 0.0 & -12880.0 & 0.0 & 64399.0 & 0.0 & -25760.0 & 0.0 & 12880. & 0.0 & 32199.0 & 0.0 \\
0.0 & 2060.8 & 0.0 & -2934.9 & 0.0 & 4434.0 & 0.0 & -2060.8 & 0.0 & -3247.4 & 0.0 & -1342.8 \\
\hline
-4.4306E5 & 0.0 & -8.5178E5 & 0.0 & -25760.0 & 0.0 & 4.4306E5 & 0.0 & 8.5178E5 & 0.0 & -25760.0 & 0.0 \\
0.0 & -2747.7 & 0.0 & 4121.5 & 0.0 & -2060.8 & 0.0 & 2747.7 & 0.0 & 4121.5 & 0.0 & -2060.8 \\
-8.5178E5 & 0.0 & -1.7207E6 & 0.0 & 12880.0 & 0.0 & -8.5178E5 & 0.0 & 1.7207E6 & 0.0 & 12880.0 & 0.0 \\
0.0 & -4121.5 & 0.0 & 3528.2 & 0.0 & -3247.4 & 0.0 & 4121.5 & 0.0 & 8836.3 & 0.0 & -2934.9 \\
25760.0 & 0.0 & -12880.0 & 0.0 & 32199.0 & 0.0 & -25760.0 & 0.0 & 12880.0 & 0.0 & 64399.0 & 0.0 \\
0.0 & 2060.8 & 0.0 & -3247.4 & 0.0 & -1342.8 & 0.0 & -2060.8 & 0.0 & -2934.9 & 0.0 & 4434.0
\end{array}\right] &
\begin{array}{c} \\ \\ 4 \\ \\ \\ \\ \\ \\ 3 \\ \\ \\ \\ \end{array}
\end{array}$$

The global stiffness matrix:

	1						2						3						4						
	024E7	0	0	0	0	0	-.24E7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	3840.0	0	0	0	5760.0	0	-3840.0	0	0	0	5760.0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	24000.	0	-36000.	0	0	0	-24000.	0	-36000.	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	0	0	0	3316.7	0	0	0	0	0	-3316.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	-36000.	0	72000.	0	0	0	36000.	0	36000.	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	5760.0	0	0	0	11520.	0	-5760.0	0	0	0	5760.0	0	0	0	0	0	0	0	0	0	0	0	0	
	-2.4E6	0	0	0	0	0	2.404E6	0	0	0	0	-5760.0	-3840.0	0	0	0	0	-5760.0	0	0	0	0	0	0	
	0	-3840.0	0	0	0	-5760.0	0	2.404E6	0	0	0	-5760.0	0	-2.4E6	0	0	0	0	0	0	0	0	0	0	
	0	0	-24000.	0	36000.	0	0	0	48000.	36000.	36000.	0	0	0	-24000.	36000.	0	0	0	0	0	0	0	0	2
	0	0	0	-3316.7	0	0	0	0	36000.	75317.	0	0	0	0	-36000.	36000.	0	0	0	0	0	0	0	0	
	0	0	-36000.	0	36000.	0	0	0	36000.	0	75317.	0	0	0	0	0	-3316.7	0	0	0	0	0	0	0	
K=	0	5760.0	0	0	0	5760.0	-5760.0	-5760.0	0	0	0	23040.	5760.0	0	0	0	0	5760.0	0	0	0	0	0	0	
	0	0	0	0	0	0	-3840.0	0	0	0	0	5760.0	4.469E5	0	8.518E5	0	-25760.	5760.0	-4.43E5	0	-8.52E5	0	-25760.	0	
	0	0	0	0	0	0	0	-2.4E6	0	0	0	0	0	2.403E6	0	4121.5	0	-2060.8	0	-2747.7	0	4121.5	0	-2060.8	
	0	0	0	0	0	0	0	0	-24000.	-36000.	0	0	8.518E5	0	1.745E6	-36000.	12880.	0	-8.52E5	0	-1.72E6	0	12880.	0	3
	0	0	0	0	0	0	0	0	36000.	36000.	0	0	0	4121.5	-36000.	80836.	0	-2934.9	0	-4121.5	0	3528.2	0	-3247.4	
	0	0	0	0	0	0	0	0	0	0	-3316.7	0	-25760.	0	12880.	0	67715.	0	25760.	0	-12880.	0	32199.	0	
	0	0	0	0	0	0	-5760.0	0	0	0	0	5760.0	5760.0	-2060.8	0	-2934.9	0	15954.	0	2060.8	0	-3247.4	0	-1342.8	
	0	0	0	0	0	0	0	0	0	0	0	0	-4.43E5	0	-8.52E5	0	25760.	0	4.431E5	0	8.518E5	0	25760.	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	-2747.7	0	-4121.5	0	2060.8	0	2747.7	0	-4121.5	0	2060.8	
	0	0	0	0	0	0	0	0	0	0	0	0	-8.52E5	0	-1.72E6	0	-12880.	0	8.518E5	0	1.721E6	0	-12880.	0	4
	0	0	0	0	0	0	0	0	0	0	0	0	0	4121.5	0	3528.2	0	-3247.4	0	-4121.5	0	8836.3	0	-2934.9	
	0	0	0	0	0	0	0	0	0	0	0	0	-25760.	0	12880.	0	32199.	0	25760.	0	-12880.	0	64399.	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	-2060.8	0	-3247.4	0	-1342.8	0	2060.8	0	-2934.9	0	4434.0	

The global stiffness matrix after taking into consideration boundary conditions:

$$
\begin{array}{cc}
 & \begin{array}{cccc} \qquad\qquad\qquad 1 \qquad\qquad\qquad & \qquad\qquad\qquad 2 \qquad\qquad\qquad & \qquad\qquad\qquad 3 \qquad\qquad\qquad & \qquad\qquad\qquad 4 \qquad\qquad\qquad \end{array} \\
\mathbf{K}^r = & \left[\begin{array}{cccccc|cccccc|cccccc|cccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 2.404E6 & 0 & 0 & 0 & 0 & -5760.0 & -3840.0 & 0 & 0 & 0 & 0 & -5760.0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.404E6 & 0 & 0 & 0 & -5760.0 & 0 & -2.4E6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 48000. & 36000. & 36000. & 0 & 0 & 0 & -24000. & 36000. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 36000. & 75317. & 0 & 0 & 0 & 0 & -36000. & 36000. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 36000. & 0 & 75317. & 0 & 0 & 0 & 0 & 0 & -3316.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -5760.0 & -5760.0 & 0 & 0 & 0 & 23040. & 5760.0 & 0 & 0 & 0 & 0 & 5760.0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & -3840.0 & 0 & 0 & 0 & 0 & 5760.0 & 4.469E5 & 0 & 8.518E5 & 0 & -25760. & 5760.0 & 0 & 0 & 0 & 0 & -25760. & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.4E6 & 0 & 0 & 0 & 0 & 0 & 2.403E6 & 0 & 4121.5 & 0 & -2060.8 & 0 & 0 & 0 & 4121.5 & 0 & -2060.8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -24000. & -36000. & 0 & 0 & 8.518E5 & 0 & 1.745E6 & -36000. & 12880. & 0 & 0 & 0 & 0 & 0 & 12880. & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 36000. & 36000. & 0 & 0 & 0 & 4121.5 & -36000. & 80836. & 0 & -2934.9 & 0 & 0 & 0 & 3528.2 & 0 & -3247.4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3316.7 & 0 & -25760. & 0 & 12880. & 0 & 67715. & 0 & 0 & 0 & 0 & 0 & 32199. & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -5760.0 & 0 & 0 & 0 & 0 & 5760.0 & 5760.0 & -2060.8 & 0 & -2934.9 & 0 & 15954. & 0 & 0 & 0 & -3247.4 & 0 & -1342.8 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4121.5 & 0 & 3528.2 & 0 & -3247.4 & 0 & 0 & 0 & 8836.3 & 0 & -2934.9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -25760. & 0 & 12880. & 0 & 32199. & 0 & 0 & 0 & 0 & 0 & 64399. & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2060.8 & 0 & -3247.4 & 0 & -1342.8 & 0 & 0 & 0 & -2934.9 & 0 & 4434.0
\end{array}\right]
\begin{array}{c} \\ \\ 1 \\ \\ \\ \\ \\ \\ 2 \\ \\ \\ \\ \\ \\ 3 \\ \\ \\ \\ \\ \\ 4 \\ \\ \\ \\ \end{array}
\end{array}
$$

The solution of this set gives the values of nodal displacements for the frame written in the vector **u** and after operation (2.75) we obtain the values of reactions of constraint supports written in the vector **r**. The components of the vectors **u** and **r** are following:

$$\mathbf{u}=\begin{array}{c}\left[\begin{array}{c}0.0\\0.0\\0.0\\0.0\\0.0\\0.0\\\hline 0.25871E-5\\-0.24441E-2\\-0.48541E-2\\-0.30158E-2\\0.27151E-2\\-0.56083E-2\\\hline 0.28192E-1\\-0.24471E-2\\-0.14110E-1\\-0.32587E-2\\0.89697E-2\\-0.12107E-1\\\hline 0.0\\0.0\\0.0\\-0.71778E-2\\0.96139E-2\\-0.13522E-1\end{array}\right]\end{array}\begin{array}{c}\\\\1\\\\\\\\\\\\2\\\\\\\\\\\\3\\\\\\\\\\\\4\\\\\\\end{array}\qquad \mathbf{r}=\left[\begin{array}{c}-6.2091\\-52.918\\18.752\\10.002\\-77.001\\-40.726\\\hline ------\\------\\------\\------\\------\\------\\\hline ------\\------\\------\\------\\------\\------\\\hline 6.2091\\-7.0818\\26.248\\------\\------\\------\end{array}\right]\begin{array}{c}\\\\1\\\\\\\\\\\\2\\\\\\\\\\\\3\\\\\\\\\\\\4\\\\\\\end{array}$$

Components of the displacement vector transformed to the local coordinate systems of elements (the equation $\mathbf{u}'^{e}=\left(\mathbf{R}^{e}\right)^{\mathrm{T}}\mathbf{u}^{e}$) enable to calculate the values of nodal forces (equation (5.7)) and next after considering external loads internal forces of elements. The values of internal forces (N, T_y, T_z, M_x, M_y, M_z) calculated in this way are given in Tab.5.E1.4.

These values are applied to drawing the graphs of both shearing forces presented in Fig.5.E1.2 and Fig.5.E1.3 and bending moments presented in Fig.5.E1.4 and Fig.5.E1.5 whereas Fig.5.E1.6 presents the deformation of the frame (displacements of elements).

Tab.5.E1.4

No e	*x/L* [/]	*N* [kN]	T_y [kN]	T_z [kN]	M_x [kNm]	M_y [kNm]	M_z [kNm]
1	0.00	6.2091	52.918	-18.752	-10.002	77.001	40.726
	0.25	6.2091	52.918	-18.752	-10.002	62.937	1.0369
	0.50	6.2091	52.918	-18.752	-10.002	48.873	-38.652
	0.50		-7.0818				
	0.75	6.2091	-7.0818	-18.752	-10.002	34.808	-33.341
	1.00	6.2091	-7.0818	-18.752	-10.002	20.744	-28.029
2	0.00	-7.0818	-6.2091	-18.752	20.744	10.002	-28.029
	0.25	-7.0818	-6.2091	-7.5023	20.744	.15677	-23.372
	0.50	-7.0818	-6.2091	3.7477	20.744	-1.2512	-18.716
	0.75	-7.0818	-6.2091	14.998	20.744	5.7783	-14.059
	1.00	-7.0818	-6.2091	26.248	20.744	21.245	-9.4019
3	0.00	-26.253	7.0818	-6.1847	0.0	0.0	0.0
	0.25	-26.253	7.0818	-6.1847	0.0	-5.1860	-5.9382
	0.50	-26.253	7.0818	-6.1847	0.0	-10.372	-11.8764
	0.50				17.910		-2.921
	0.75	-26.253	7.0818	-6.1847	17.910	-15.558	-8.8595
	1.00	-26.253	7.0818	-6.1847	17.910	-20.744	-14.798

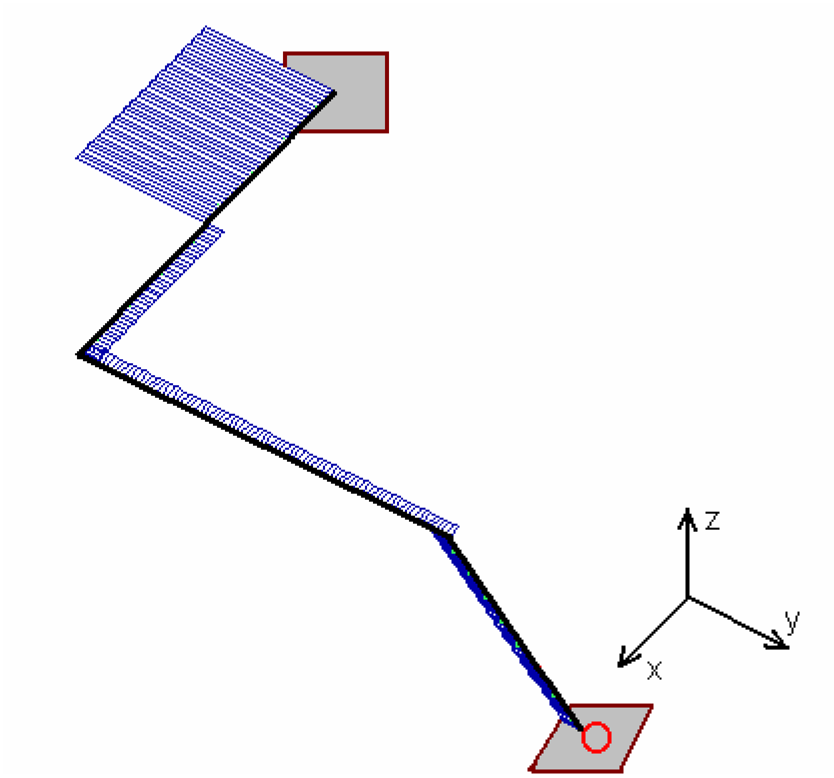

Fig.5.E1.2. The graph of shearing forces T_y.

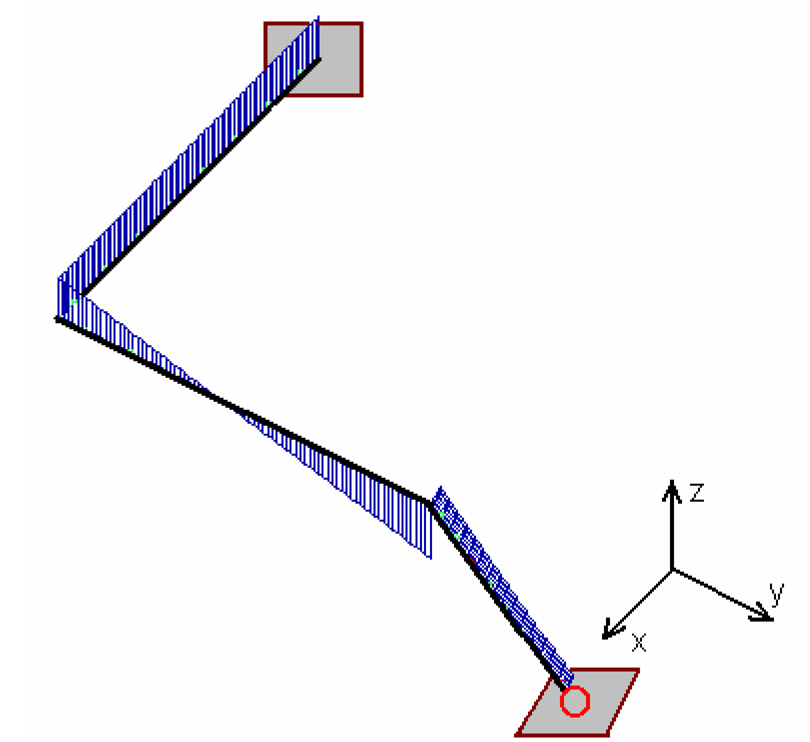

Fig.5.E1.3. The graph of shearing forces T_z.

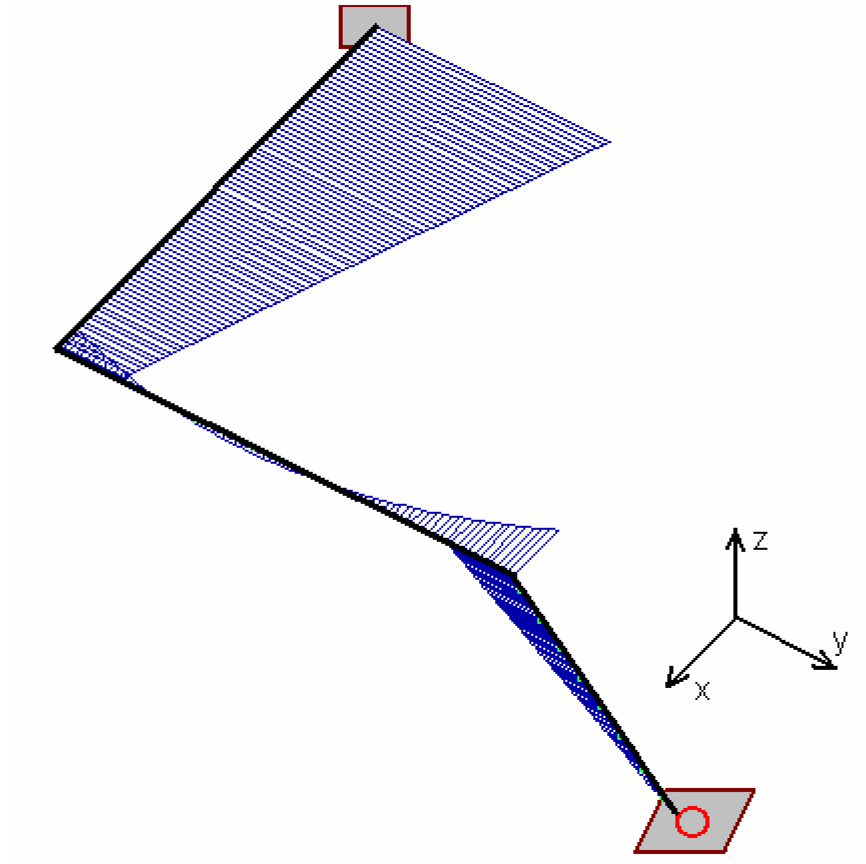

Fig.5.E1.4. The graph of bending moments M_y.

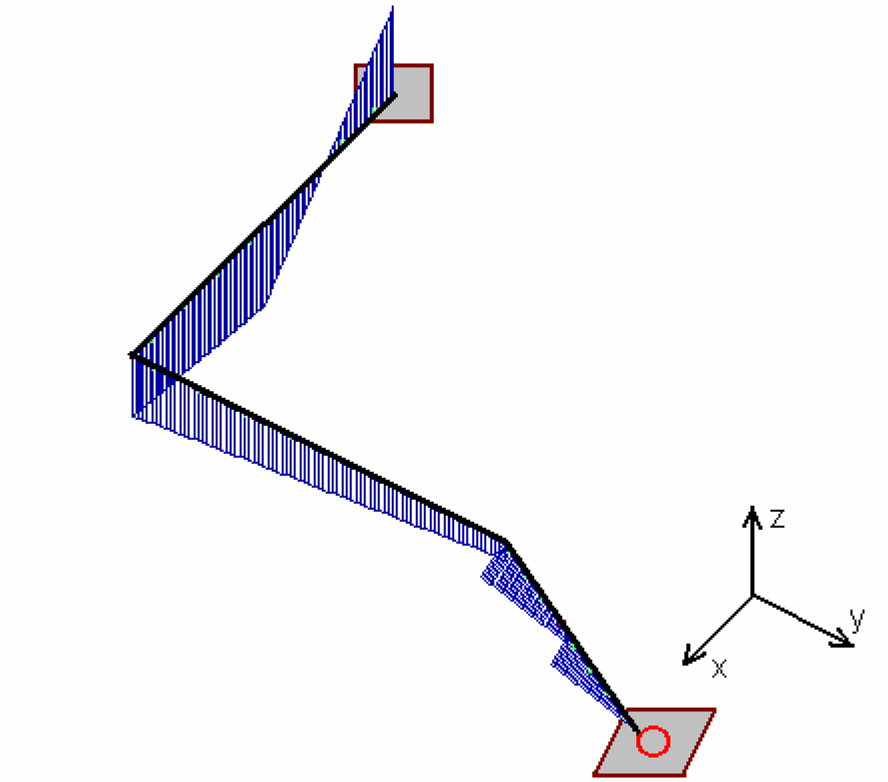

Fig.5.E1.5. The graph of bending moments M_z.

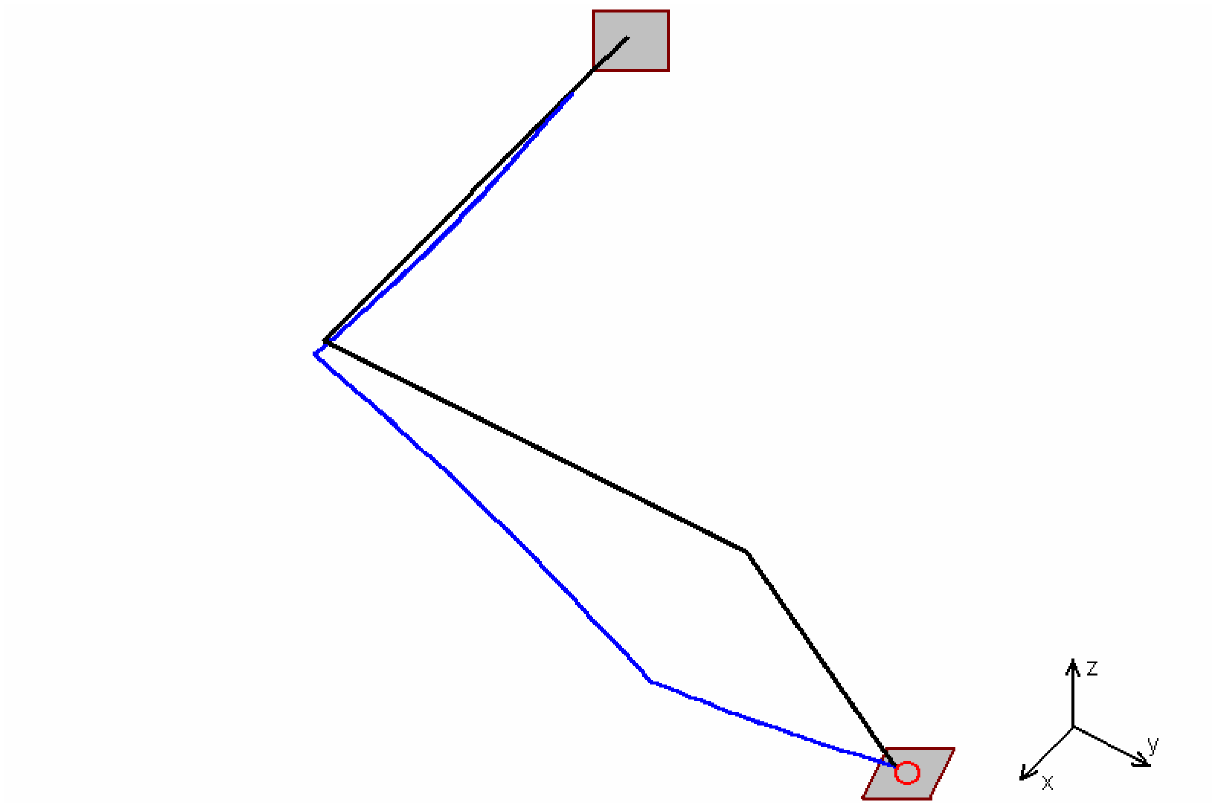

Fig.5.E1.6. The scheme of the deformed frame.