Example 2.E5.

The content of the example

A 2D truss loaded with forces acting on nodes No 2 and No 5 with the geometry analogous to that analysed in example 2.E1 rests on a moveable articulated support at node No 1 and on an elastic support with stiffness h_{6Y}=50 kN/m at node No 6 (Fig.2.E5.1).

Calculate nodal displacements and internal forces in the elements.

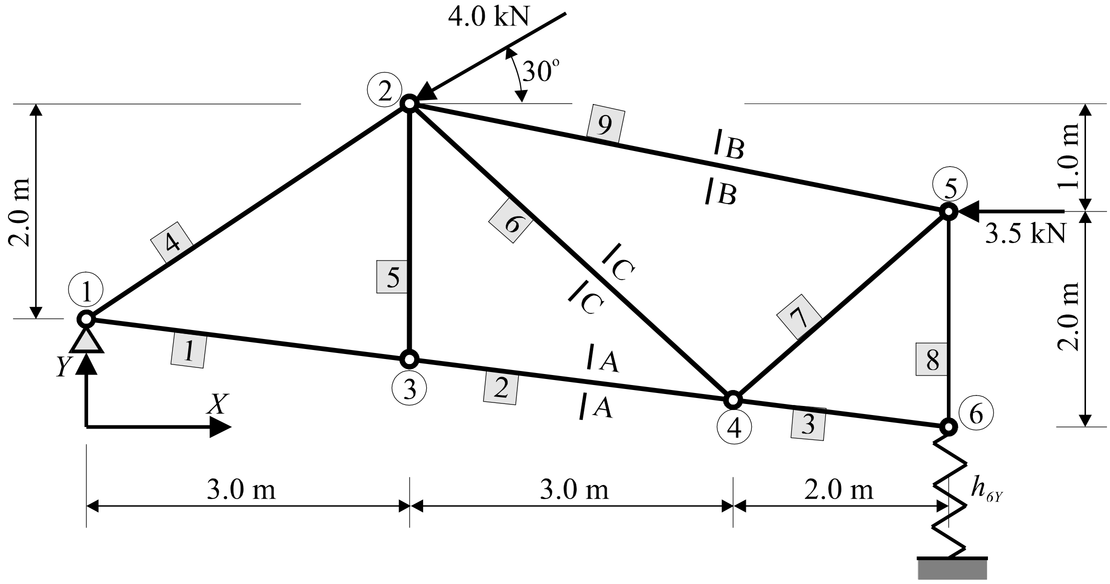

Fig.2.E5.1

The solution of the example

Similar structure geometry and load allow to use matrices **K**, **p**, $\mathbf{p}^r$ formed in example 2.E1. The boundary conditions require satisfying the equations:

$u_{1X} = 0$, $u_{1Y} = 0$, $R_{6Y}=-h_{6Y}\, u_{6Y}$.

We consider the first two boundary conditions in a standard way (comp. example 2.E1), and the third boundary condition requires the modification of a term lying on the main diagonal in row No 12 of the matrix **K** (comp. section 2.8) because No 12 is the global number of the degree of freedom u_{6Y}. The value of the component $K_{12\ 12}$ is equal to 24 229 kN/m (comp. matrix **K** from example 2.E1), hence after adding the value of support stiffness to it, we obtain $K_{12\ 12}$=24 229 kN/m+50 kN/m=24 279 kN/m.

The global stiffness matrix after taking into consideration the elastic support and boundary conditions $\mathbf{K}^r$ is presented on the next page.

The global stiffness matrix after taking into consideration boundary conditions:

$$\mathbf{K}^{r} = \begin{array}{c} \begin{array}{cccccc} \quad 1 \quad & \quad\quad 2 \quad\quad & \quad\quad 3 \quad\quad & \quad\quad 4 \quad\quad & \quad\quad 5 \quad\quad & \quad\quad 6 \quad\quad \end{array} \\ \left[\begin{array}{cc|cc|cc|cc|cc|cc} \mathit{1} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} \\ \mathit{0} & \mathit{1} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} & \mathit{0} \\ \hline \mathit{0} & \mathit{0} & 19710.1 & 3012.2 & 0 & 0 & -1442.0 & 1321.9 & -9051.5 & 1810.3 & 0 & 0 \\ \mathit{0} & \mathit{0} & 3012.2 & 10217.4 & 0 & -4547.4 & 1321.9 & -1211.7 & 1810.3 & -362.1 & 0 & 0 \\ \hline \mathit{0} & \mathit{0} & 0 & 0 & 19540.2 & -2442.5 & -9770.1 & 1221.3 & 0 & 0 & 0 & 0 \\ \mathit{0} & \mathit{0} & 0 & -4547.4 & -2442.5 & 4852.7 & 1221.3 & -152.7 & 0 & 0 & 0 & 0 \\ \hline \mathit{0} & \mathit{0} & -1442.0 & 1321.9 & -9770.1 & 1221.3 & 28169.0 & -2361.1 & -2301.7 & -2014.0 & -14655.2 & 1831.9 \\ \mathit{0} & \mathit{0} & 1321.9 & -1211.7 & 1221.3 & -152.7 & -2361.1 & 3355.6 & -2014.0 & -1762.2 & 1831.9 & -229.0 \\ \hline \mathit{0} & \mathit{0} & -9051.5 & 1810.3 & 0 & 0 & -2301.7 & -2014.0 & 11353.2 & 203.7 & 0 & 0 \\ \mathit{0} & \mathit{0} & 1810.3 & -362.1 & 0 & 0 & -2014.0 & -1762.2 & 203.7 & 26124.3 & 0 & -24000.0 \\ \hline \mathit{0} & \mathit{0} & 0 & 0 & 0 & 0 & -14655.2 & 1831.9 & 0 & 0 & 14655.2 & -1831.9 \\ \mathit{0} & \mathit{0} & 0 & 0 & 0 & 0 & 1831.9 & -229.0 & 0 & -24000.0 & -1831.9 & \boldsymbol{24279.0} \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \end{array}$$

Modifications introduced to $\mathbf{K}^r$ modifications are marked in italic fonts and the new value of the components $K_{12\,12}$ is additionally distinguished in bold font.

Further calculations are analogous to those executed in example 2.E1, thus we give only obtained results, that is, the nodal displacement vector **u** and the support reaction vector **r**:

$$\mathbf{u}=\left[\begin{array}{c} 0.0 \\ 0.0 \\ \hline -3.63791\text{E-}3 \\ 4.75295\text{E-}3 \\ \hline 3.24010\text{E-}4 \\ 4.75295\text{E-}3 \\ \hline 4.75541\text{E-}4 \\ 8.12607\text{E-}3 \\ \hline -2.62764\text{E-}3 \\ 11.09357\text{E-}3 \\ \hline 8.43595\text{E-}4 \\ 11.07051\text{E-}3 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array} \qquad \mathbf{r}=\left[\begin{array}{c} 6.964 \\ 2.554 \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ -0.554 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

The scheme of the truss deformation is presented in Fig.2.E5.2.

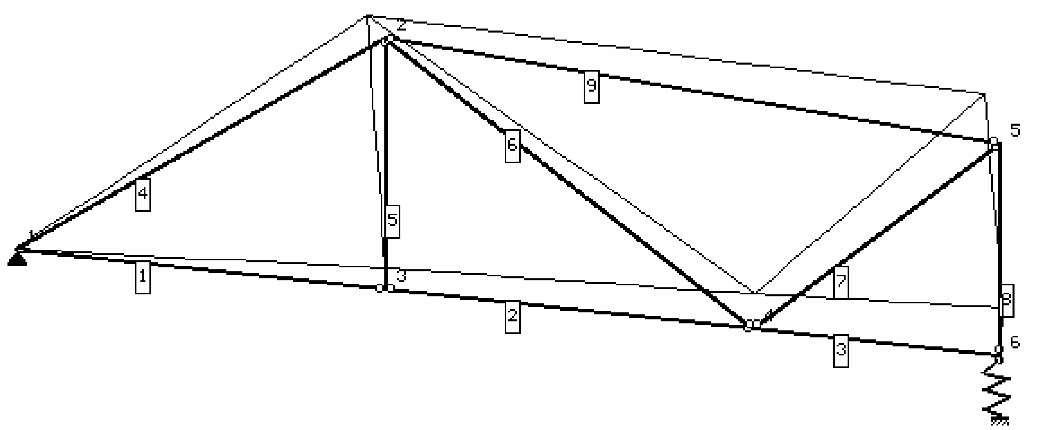

Fig.2.E5.2

We give nodal displacements, internal forces and stresses in Tab.2.E5.1.

Tab.2.E5.1

We obtain columns marked by u_{iX}, u_{iY}, u_{jX}, u_{jY} in the way shown in Fig.2.14

No n	u_{iX} [m]	u_{iY} [m]	u_{jX} [m]	u_{jY} [m]	u_{ix} [m] (2.22)	u_{jx} [m] (2.22)	N [kN] (2.76)	σ [kN/m^2] (2.77)
1	0.0	0.0	3.24010E-4	4.75295E-3	0.0	-2.68023E-4	-2.660	-1063.8
2	3.24010E-4	4.75295E-3	4.75541E-4	8.12607E-3	-2.68023E-4	-5.36046E-4	-2.660	-1063.8
3	4.75541E-4	8.12607E-3	8.43595E-4	11.07051E-3	-5.36047E-4	-5.36048E-4	0.0	0.0
4	0.0	0.0	-3.63791E-3	4.75295E-3	0.0	-3.90462E-4	-5.198	-1299.5
5	3.24010E-4	4.75295E-3	-3.63791E-3	4.75295E-3	4.75295E-3	4.75295E-3	0.0	0.0
6	-3.63791E-3	4.75295E-3	4.75541E-4	8.12607E-3	-5.89338E-3	-5.14043E-3	1.998	2220.1
7	4.75541E-4	8.12607E-3	-2.62764E-3	11.09357E-3	5.70893E-3	5.32766E-3	-1.549	-1721.6
8	8.43595E-4	11.07051E-3	-2.62764E-3	11.09357E-3	1.10705E-2	1.10936E-2	0.553	138.4
9	-3.63791E-3	4.75295E-3	-2.62764E-3	11.09357E-3	-4.49939E-3	-4.75224E-3	-2.380	-595.0