Example 6.E1.

The content of the example

A two-dimensional pentagonal plate (shortly called a plate) with thickness of 5 cm and with the scheme shown in Fig.6.E1.1 is loaded with concentrated forces having the values of 5.0 kN and 10.0 kN and the temperature increment Δt=50 K. The material which the structure is made from is characterised by constants:

- Young's modulus $E=2\cdot10^7$ kPa,
- Poisson's ratio $\nu=0.25$.

Determine nodal displacements of the structure, constraint supports and stresses in elements.

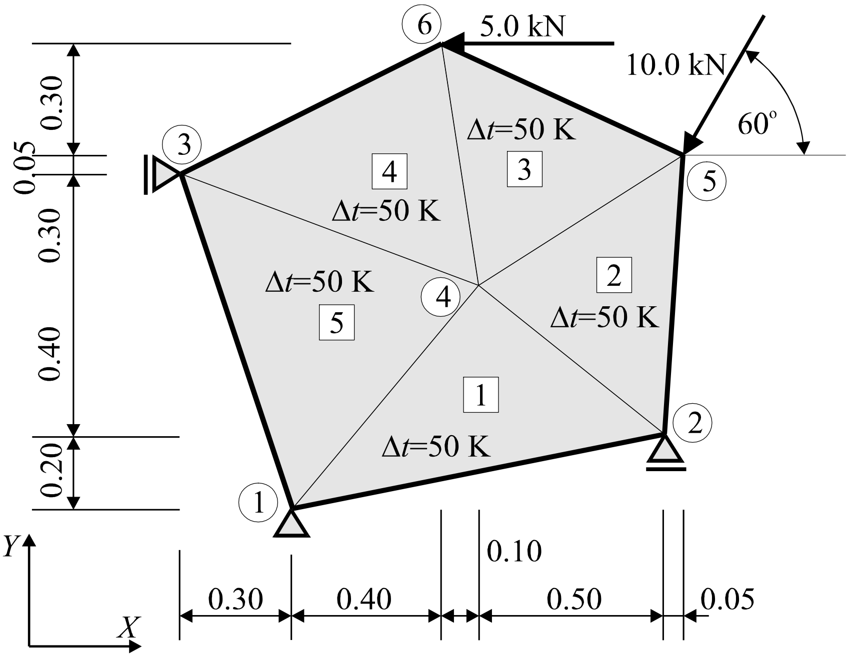

Fig.6.E1.1

The solution of the example

Since the thickness of the structure is small in comparison with its other dimensions, it enables to assume plane stress inside the plate. We start solving the problem from the discretization of the structure, that is, the division of the plate into finite elements. As we are more anxious to show the FEM algorithm and we do not care so much about the exactness of obtained results, we divide the plate only into five triangular elements because in this way it will be easier for the reader to follow all the stages of the problem. In practice when we want to obtain exact results, the plate should be divided into a few hundreds of elements. Tab.6.E1.1

contains nodal coordinates of the model and the values of concentrated forces acting on nodes, whereas Tab.6.E1.2 contains data defining elements: global numbers of the nodes i, j, k and areas of the elements.

Tab.6.E1.1

Node No n	X_n [m]	Y_n [m]	P_{Xn} [kN]	P_{Yn} [kN]
1	0.3	0.0	-----	-----
2	1.3	0.2	-----	-----
3	0.0	0.9	-----	-----
4	0.8	0.6	-----	-----
5	1.35	0.95	-5.0	-8.66026
6	0.7	1.25	-5.0	-----

Tab.6.E1.2

Element No n	Node No n_i	 n_j	 n_k	A_n [m^2]
1	1	2	4	0.25000
2	2	5	4	0.19750
3	4	5	6	0.19625
4	3	4	6	0.24500
5	1	4	3	0.31500

First, we have to determine shape functions of elements. We choose CST elements described by linear shape functions (equation (6.14)), so we have to determine at least six coefficients of these functions: b_i, b_j, b_k, c_i, c_j, c_k , and thus three coefficients a_i, a_j, a_k are dispensable for further consideration. For this purpose we use equations (6.18). Here we show determining coefficients of shape function N_1^1, N_2^1, N_4^1 for element No 1 only:

$$W = \begin{vmatrix} 1 & 0.3\text{ m} & 0.0\text{ m} \\ 1 & 1.3\text{ m} & 0.2\text{ m} \\ 1 & 0.8\text{ m} & 0.6\text{ m} \end{vmatrix} = 0.5\text{ m}^2,$$

$$W_{b_1} = \begin{vmatrix} 1 & 1 & 0.0\text{ m} \\ 1 & 0 & 0.2\text{ m} \\ 1 & 0 & 0.6\text{ m} \end{vmatrix} = 0.4\text{ m}, \qquad b_1 = \frac{W_{b_1}}{W} = \frac{0.4\text{ m}}{0.5\text{ m}^2} = 0.8/\text{m}$$

$$W_{c_1} = \begin{vmatrix} 1 & 0.3\text{ m} & 1 \\ 1 & 1.3\text{ m} & 0 \\ 1 & 0.8\text{ m} & 0 \end{vmatrix} = -0.5\text{ m}, \qquad c_1 = \frac{W_{c_1}}{W} = \frac{-0.5\text{ m}}{0.5\text{ m}^2} = -1.0/\text{m}$$

$$N_1^1 = a_1 + x \cdot 0.8/\text{m} - \text{y} \cdot 1.0/\text{m}$$

$$W_{b_2} = \begin{vmatrix} 1 & 0 & 0.0\text{ m} \\ 1 & 1 & 0.2\text{ m} \\ 1 & 0 & 0.6\text{ m} \end{vmatrix} = 0.6\text{ m}, \qquad b_2 = \frac{W_{b_2}}{W} = \frac{0.6\text{ m}}{0.5\text{ m}^2} = 1.2/\text{m}$$

$$W_{c_2} = \begin{vmatrix} 1 & 0.3\text{ m} & 0 \\ 1 & 1.3\text{ m} & 1 \\ 1 & 0.8\text{ m} & 0 \end{vmatrix} = -0.5\text{ m}, \qquad c_2 = \frac{W_{c_2}}{W} = \frac{-0.5\text{ m}}{0.5\text{ m}^2} = -1.0/\text{m}$$

$$N_2^1 = a_2 + x \cdot 1.2/\text{m} - \text{y} \cdot 1.0/\text{m}$$

$$W_{b_4} = \begin{vmatrix} 1 & 0 & 0.0\text{ m} \\ 1 & 0 & 0.2\text{ m} \\ 1 & 1 & 0.6\text{ m} \end{vmatrix} = -0.2\text{ m}, \qquad b_4 = \frac{W_{b_4}}{W} = \frac{0.2\text{ m}}{0.5\text{ m}^2} = -0.4/\text{m}$$

$$W_{c_4} = \begin{vmatrix} 1 & 0.3\text{ m} & 0 \\ 1 & 1.3\text{ m} & 0 \\ 1 & 0.8\text{ m} & 1 \end{vmatrix} = 1.0\text{ m}, \qquad c_4 = \frac{W_{c_4}}{W} = \frac{1.0\text{ m}}{0.5\text{ m}^2} = 2.0/\text{m}$$

$$N_4^1 = a_4 - x \cdot 0.4/\text{m} + \text{y} \cdot 2.0/\text{m}$$

We can determine displacements of a point with any coordinates x and y and which lies on element No 1 on the basis of the following equations (comp. equations (6.11)):

$$u_x(x,y) = N_1^1(x,y) \cdot 0\text{ m} + N_2^1(x,y) \cdot 5.467 \cdot 10^{-3}\text{ m} + N_4^1(x,y) \cdot 3.421 \cdot 10^{-3}\text{ m}$$

$$u_y(x,y) = N_1^1(x,y)\cdot 0\ \mathrm{m} + N_2^1(x,y)\cdot 0\ \mathrm{m} + N_4^1(x,y)\cdot 2.458\cdot 10^{-3}\ \mathrm{m}$$

Similarly, we can determine other coefficients of shape functions changing only nodal coordinates in the equations presented on the previous page. Tab.6.E1.3 contains values of all coefficients of shape functions for five elements.

Tab.6.E1.3

No n	b_i [1/m]	c_i [1/m]	b_j [1/m]	c_j [1/m]	b_k [1/m]	c_k [1/m]
1	-0.80	-1.00	1.20	-1.00	-0.40	2.00
2	0.89	-1.39	1.01	1.27	-1.90	0.13
3	-0.76	-1.66	1.66	0.25	-0.89	1.40
4	-1.33	-0.20	0.71	-1.43	0.61	1.63
5	-0.48	-1.27	1.43	0.48	-0.95	0.79

If we know shape functions of elements, we can determine nodal force vectors $\mathbf{f}^{et}$ caused by a temperature load. We apply equation (6.46) to calculate components of these vectors:

$$\mathbf{f}^{1t} = \left[\begin{array}{c} 1333.3 \\ 1666.7 \\ \hline -2000 \\ 1666.7 \\ \hline 666.67 \\ -3333.3 \end{array}\right] \begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 4 \end{array} \qquad \mathbf{f}^{2t} = \left[\begin{array}{c} -1166.7 \\ 1833.3 \\ \hline -1333.3 \\ -1666.7 \\ \hline 2500 \\ -166.67 \end{array}\right] \begin{array}{c} \\ 2 \\ \\ 5 \\ \\ 4 \end{array}$$

$$\mathbf{f}^{3t} = \left[\begin{array}{c} 1000 \\ 2166.7 \\ \hline -2166.7 \\ -333.33 \\ \hline 1166.7 \\ -1833.3 \end{array}\right] \begin{array}{c} \\ 4 \\ \\ 5 \\ \\ 6 \end{array} \qquad \mathbf{f}^{4t} = \left[\begin{array}{c} 2166.7 \\ 333.33 \\ \hline -1166.7 \\ 2333.3 \\ \hline -1000 \\ -2666.7 \end{array}\right] \begin{array}{c} \\ 3 \\ \\ 4 \\ \\ 6 \end{array}$$

$$\mathbf{f}^{5t} = \left[\begin{array}{c} 1000 \\ 2666.7 \\ \hline -3000 \\ -1000 \\ \hline 2000 \\ -1666.7 \end{array}\right]\begin{array}{l} \\ 1 \\ \\ 4 \\ \\ 3 \end{array}$$

After considering concentrated forces acting on nodes, we obtain the global vector of nodal forces:

$$\mathbf{p} = \mathbf{p}^{o} - \sum_{e=1}^{5} \mathbf{p}^{et} \; .$$

The vector **p** is presented on the next page.

The global vector of nodal forces:

$$
\mathbf{p}=\left[\begin{array}{c} 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline -5.0 \\ -8.66026 \\ \hline -5.0 \\ 0.0 \end{array}\right]
-\left[\begin{array}{c} 1333.3 \\ 1666.7 \\ \hline -2000 \\ 1666.7 \\ \hline \\ \\ \hline 666.67 \\ -3333.3 \\ \hline \\ \\ \hline \\ \\ \end{array}\right]
-\left[\begin{array}{c} \\ \\ \hline -1166.7 \\ 1833.3 \\ \hline \\ \\ \hline 2500 \\ -166.67 \\ \hline -1333.3 \\ -1666.7 \\ \hline \\ \\ \end{array}\right]
-\left[\begin{array}{c} \\ \\ \hline \\ \\ \hline \\ \\ \hline 1000 \\ 2166.7 \\ \hline -2166.7 \\ -333.33 \\ \hline 1166.7 \\ -1833.3 \end{array}\right]
-\left[\begin{array}{c} \\ \\ \hline \\ \\ \hline 2166.7 \\ 333.33 \\ \hline -1166.7 \\ 2333.3 \\ \hline \\ \\ \hline -1000 \\ -2666.7 \end{array}\right]
-\left[\begin{array}{c} 1000 \\ 2666.7 \\ \hline \\ \\ \hline 2000 \\ -1666.7 \\ \hline -3000 \\ -1000 \\ \hline \\ \\ \hline \\ \\ \end{array}\right]
=\left[\begin{array}{c} -2333.3 \\ -4333.3 \\ \hline 3166.7 \\ -3500 \\ \hline -4166.7 \\ 1333.3 \\ \hline 0 \\ 0 \\ \hline 3495 \\ 1991.3 \\ \hline -171.67 \\ 4500 \end{array}\right]
\begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \end{array}
$$

Next we determine components of element stiffness matrices on the basis of equations (6.27) and (6.29) obtaining

$$\mathbf{K}^1=\begin{array}{cc} \begin{array}{cccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{4} \end{array} & \\ \left[\begin{array}{cc|cc|cc} 270670.0 & 133330.0 & -156000.0 & -66667.0 & -114670.0 & -66667.0 \\ 133330.0 & 330670.0 & 0.0 & 170670.0 & -133330.0 & -501330.0 \\ \hline -156000.0 & 0.0 & 484000.0 & -200000.0 & -328000.0 & 200000.0 \\ -66667.0 & 170670.0 & -200000.0 & 410670.0 & 266670.0 & -581330.0 \\ \hline -114670.0 & -133330.0 & -328000.0 & 266670.0 & 442670.0 & -133330.0 \\ -66667.0 & -501330.0 & 200000.0 & -581330.0 & -133330.0 & 1082700.0 \end{array}\right] & \begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 4 \end{array} \end{array}$$

$$\mathbf{K}^2=\begin{array}{cc} \begin{array}{cccccc} \multicolumn{2}{c}{2} & \multicolumn{2}{c}{5} & \multicolumn{2}{c}{4} \end{array} & \\ \left[\begin{array}{cc|cc|cc} 318570.0 & -162450.0 & 49789.0 & -52321.0 & -368350.0 & 214770.0 \\ -162450.0 & 470460.0 & 14346.0 & -300420.0 & 148100.0 & -170040.0 \\ \hline 49789.0 & 14346.0 & 342620.0 & 168780.0 & -392410.0 & -183120.0 \\ -52321.0 & -300420.0 & 168780.0 & 418570.0 & -116460.0 & -118140.0 \\ \hline -368350.0 & 148100.0 & -392410.0 & -116460.0 & 760760.0 & -31646.0 \\ 214770.0 & -170040.0 & -183120.0 & -118140.0 & -31646.0 & 288190.0 \end{array}\right] & \begin{array}{c} \\ 2 \\ \\ 5 \\ \\ 4 \end{array} \end{array}$$

$$\mathbf{K}^3=\begin{array}{cc} \begin{array}{cccccc} \multicolumn{2}{c}{4} & \multicolumn{2}{c}{5} & \multicolumn{2}{c}{6} \end{array} & \\ \left[\begin{array}{cc|cc|cc} 337580.0 & 165610.0 & -298090.0 & -225480.0 & -39490.0 & 59873.0 \\ 165610.0 & 619960.0 & -158810.0 & -187690.0 & -6794.1 & -432270.0 \\ \hline -298090.0 & -158810.0 & 579190.0 & 55202.0 & -281100.0 & 103610.0 \\ -225480.0 & -187690.0 & 55202.0 & 228870.0 & 170280.0 & -41189.0 \\ \hline -39490.0 & -6794.1 & -281100.0 & 170280.0 & 320590.0 & -163480.0 \\ 59873.0 & -432270.0 & 103610.0 & -41189.0 & -163480.0 & 473460.0 \end{array}\right] & \begin{array}{c} \\ 4 \\ \\ 5 \\ \\ 6 \end{array} \end{array}$$

$$\mathbf{K}^4=\begin{array}{cc} \begin{array}{cccccc} \multicolumn{2}{c}{3} & \multicolumn{2}{c}{4} & \multicolumn{2}{c}{6} \end{array} & \\ \left[\begin{array}{cc|cc|cc} 463950.0 & 44218.0 & -219050.0 & 109520.0 & -244900.0 & -153740.0 \\ 44218.0 & 183330.0 & 176190.0 & -16667.0 & -220410.0 & -166670.0 \\ \hline -219050.0 & 176190.0 & 333330.0 & -166670.0 & -114290.0 & -9523.8 \\ 109520.0 & -16667.0 & -166670.0 & 583330.0 & 57143.0 & -566670.0 \\ \hline -244900.0 & -220410.0 & -114290.0 & 57143.0 & 359180.0 & 163270.0 \\ -153740.0 & -166670.0 & -9523.8 & -566670.0 & 163270.0 & 733330.0 \end{array}\right] & \begin{array}{c} \\ 3 \\ \\ 4 \\ \\ 6 \end{array} \end{array}$$

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \quad 1 & & \quad 4 & & \quad 3 & \end{array} \\
\mathbf{K}^5= & \left[\begin{array}{cc|cc|cc}
279370.0 & 126980.0 & -304760.0 & -247620.0 & 25397.0 & 120630.0 \\
126980.0 & 570370.0 & -180950.0 & -288890.0 & 53968.0 & -281480.0 \\
\hline
-304760.0 & -180950.0 & 714290.0 & 142860.0 & -409520.0 & 38095.0 \\
-247620.0 & -288890.0 & 142860.0 & 333330.0 & 104760.0 & -44444.0 \\
\hline
25397.0 & 53968.0 & -409520.0 & 104760.0 & 384130.0 & -158730.0 \\
120630.0 & -281480.0 & 38095.0 & -44444.0 & -158730.0 & 325930.0
\end{array}\right]
\begin{array}{c} \\ 1 \\ \\ 4 \\ \\ 3 \end{array}
\end{array}
$$

The aggregation of the global stiffness matrix is done in accordance with the equation:

$$
\begin{array}{cc}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\mathbf{K}= & \left[\begin{array}{c|c|c|c|c|c}
K^1_{11}+K^5_{11} & K^1_{12} & K^5_{13} & K^1_{14}+K^5_{14} & & \\
\hline
 & K^1_{22}+K^2_{22} & & K^1_{24}+K^2_{24} & K^2_{25} & \\
\hline
K^5_{31} & & K^4_{33}+K^5_{33} & K^4_{34}+K^5_{34} & & K^4_{36} \\
\hline
K^1_{41}+K^5_{41} & K^1_{42}+K^2_{42} & K^4_{43}+K^5_{43} & K^1_{44}+K^2_{44}+K^3_{44}+K^4_{44}+K^5_{44} & K^2_{45}+K^3_{45} & K^3_{46}+K^4_{46} \\
\hline
 & K^2_{52} & & K^2_{54}+K^3_{54} & K^2_{55}+K^3_{55} & K^3_{56} \\
\hline
 & & K^4_{63} & K^3_{64}+K^4_{64} & K^3_{65} & K^3_{66}+K^4_{66}
\end{array}\right]
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\end{array}
$$

and after substituting the earlier calculated values for $\mathbf{K}^e_{ij}$, we obtain the matrix **K** (presented on the next page).

The global stiffness matrix:

$$
\mathbf{K}=\begin{array}{cc}
\begin{array}{cccccccccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} & \multicolumn{2}{c}{4} & \multicolumn{2}{c}{5} & \multicolumn{2}{c}{6} \end{array} & \\
\left[\begin{array}{cc|cc|cc|cc|cc|cc}
550030.0 & 260320.0 & -156000.0 & -66667.0 & 25397.0 & 120630.0 & -419430.0 & -314290.0 & 0 & 0 & 0 & 0 \\
260320.0 & 901040.0 & 0 & 170670.0 & 53968.0 & -281480.0 & -314290.0 & -790220.0 & 0 & 0 & 0 & 0 \\
\hline
-156000.0 & 0 & 802570.0 & -362450.0 & 0 & 0 & -696350.0 & 414770.0 & 49789.0 & -52321.0 & 0 & 0 \\
-66667.0 & 170670.0 & -362450.0 & 881130.0 & 0 & 0 & 414770.0 & -751380.0 & 14346.0 & -300420.0 & 0 & 0 \\
\hline
25397.0 & 53968.0 & 0 & 0 & 848070.0 & -114510.0 & -628570.0 & 214290.0 & 0 & 0 & -244900.0 & -153740.0 \\
120630.0 & -281480.0 & 0 & 0 & -114510.0 & 509260.0 & 214290.0 & -61111.0 & 0 & 0 & -220410.0 & -166670.0 \\
\hline
-419430.0 & -314290.0 & -696350.0 & 414770.0 & -628570.0 & 214290.0 & 2588600.0 & -23183.0 & -690490.0 & -341930.0 & -153780.0 & 50349.0 \\
-314290.0 & -790220.0 & 414770.0 & -751380.0 & 214290.0 & -61111.0 & -23183.0 & 2907500.0 & -341930.0 & -305830.0 & 50349.0 & -998940.0 \\
\hline
0 & 0 & 49789.0 & 14346.0 & 0 & 0 & -690490.0 & -341930.0 & 921810.0 & 223980.0 & -281100.0 & 103610.0 \\
0 & 0 & -52321.0 & -300420.0 & 0 & 0 & -341930.0 & -305830.0 & 223980.0 & 647440.0 & 170280.0 & -41189.0 \\
\hline
0 & 0 & 0 & 0 & -244900.0 & -220410.0 & -153780.0 & 50349.0 & -281100.0 & 170280.0 & 679780.0 & -216.7 \\
0 & 0 & 0 & 0 & -153740.0 & -166670.0 & 50349.0 & -998940.0 & 103610.0 & -41189.0 & -216.7 & 1206800.0
\end{array}\right] &
\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}
\end{array}
$$

We describe conditions of support of the plate by equations:

$u_{1X} = 0$ (1), $u_{1Y} = 0$ (2), $u_{2Y} = 0$ (4), $u_{3X} = 0$ (5),

where global numbers of degrees of freedom are given in the brackets.

The consideration of these conditions in the nodal force vector gives the following vector:

$$\mathbf{p}^r = \left[\begin{array}{c} \mathit{0} \\ \mathit{0} \\ \hline 3166.7 \\ \mathit{0} \\ \hline \mathit{0} \\ 1333.3 \\ \hline 0 \\ 0 \\ \hline 3495 \\ 1991.3 \\ \hline -171.67 \\ 4500 \end{array}\right] \begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \end{array}$$

where modified components are marked in italic fonts.

The modifications of components of the stiffness matrix leading to the consideration of supports gives the matrix $\mathbf{K}^r$ (presented on the next page) where, as in the vector $\mathbf{p}^r$, modified components are marked in italic fonts.

The global stiffness matrix shown after taking into consideration boundary conditions:

$$
\mathbf{K}^{r} = \begin{array}{cc}
\begin{array}{cccccccccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} & \multicolumn{2}{c}{4} & \multicolumn{2}{c}{5} & \multicolumn{2}{c}{6} \end{array} & \\
\left[\begin{array}{cc|cc|cc|cc|cc|cc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 802570.0 & 0 & 0 & 0 & -696350.0 & 414770.0 & 49789.0 & -52321.0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 509260.0 & 214290.0 & -61111.0 & 0 & 0 & -220410.0 & -166670.0 \\
\hline
0 & 0 & -696350.0 & 0 & 0 & 214290.0 & 2588600.0 & -23183.0 & -690490.0 & -341930.0 & -153780.0 & 50349.0 \\
0 & 0 & 414770.0 & 0 & 0 & -61111.0 & -23183.0 & 2907500.0 & -341930.0 & -305830.0 & 50349.0 & -998940.0 \\
\hline
0 & 0 & 49789.0 & 0 & 0 & 0 & -690490.0 & -341930.0 & 921810.0 & 223980.0 & -281100.0 & 103610.0 \\
0 & 0 & -52321.0 & 0 & 0 & 0 & -341930.0 & -305830.0 & 223980.0 & 647440.0 & 170280.0 & -41189.0 \\
\hline
0 & 0 & 0 & 0 & 0 & -220410.0 & -153780.0 & 50349.0 & -281100.0 & 170280.0 & 679780.0 & -216.7 \\
0 & 0 & 0 & 0 & 0 & -166670.0 & 50349.0 & -998940.0 & 103610.0 & -41189.0 & -216.7 & 1206800.0
\end{array}\right] &
\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}
\end{array}
$$

Solving the set of equations:

$$\mathbf{K}^r\,\mathbf{u}=\mathbf{p}^r$$

gives us values of displacements vector **u** and after inserting this vector into equation (2.75), we obtain vector **r** of support reactions:

$$\mathbf{u}=\left[\begin{array}{c} 0.0\\ 0.0\\ \hline 5.467\text{E-}03\\ 0.0\\ \hline 0.0\\ 5.047\text{E-}03\\ \hline 3.421\text{E-}03\\ 2.458\text{E-}03\\ \hline 6.606\text{E-}03\\ 3.570\text{E-}03\\ \hline 3.815\text{E-}03\\ 5.873\text{E-}03 \end{array}\right]\begin{array}{c} 1\\ \\ 2\\ \\ 3\\ \\ 4\\ \\ 5\\ \\ 6\\ \\ \end{array}\qquad \mathbf{r}=\left[\begin{array}{c} -117.86\\ -104.51\\ \hline -----\\ 113.17\\ \hline 127.86\\ -----\\ \hline -----\\ -----\\ \hline -----\\ -----\\ \hline -----\\ ----- \end{array}\right]\begin{array}{c} 1\\ \\ 2\\ \\ 3\\ \\ 4\\ \\ 5\\ \\ 6\\ \\ \end{array}$$

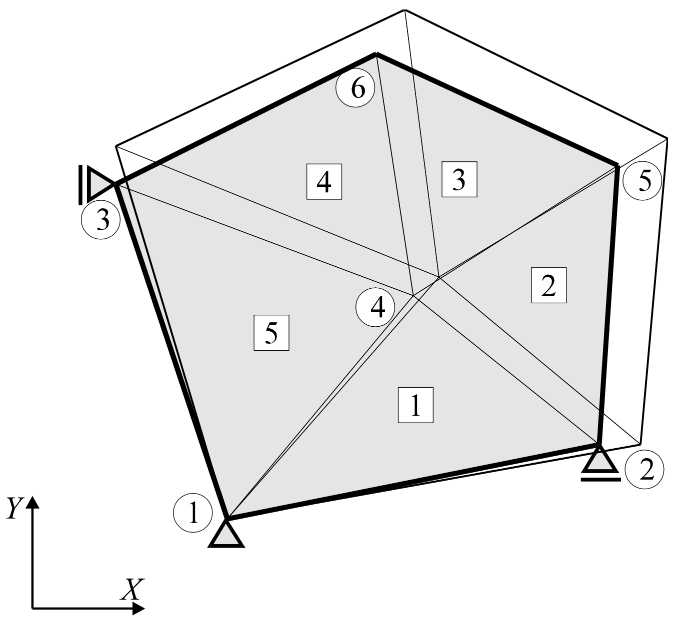

Fig.6.E1.2. The scheme of the deformed plate.

Fig.6.E1.2 shows displacements of the plate.

Now we calculate element strains from equations (6.21) and (6.22) and next after taking into consideration plane stress, we determine components of the stress vector from equation (6.5) taking into account elastic constants described by equation (1.13). Fig.6.E1.3 shows the map of values of components of the stress vector σ_x obtained on the basis of the mentioned equations. Stresses at nodes are average values calculated from the values for elements touching suitable nodes.

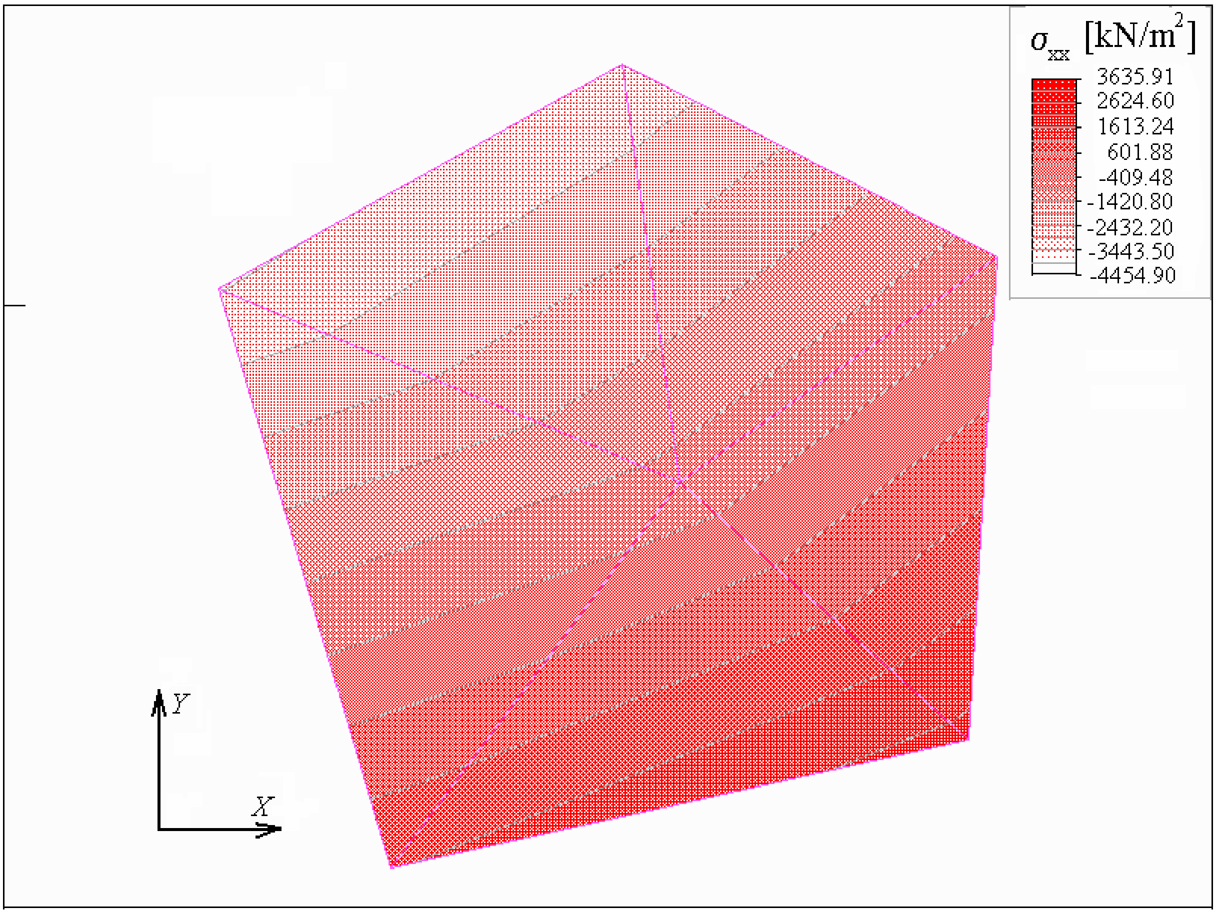

Fig.6.E1.3. The map of values of the direct stress σ_x.