Example 2.E4.

The content of the example

The truss with the geometry and static scheme analogous to those analysed in the example 2.E2 is loaded with the forced displacement of node No 1 (Fig.2.E4.1). The components of the displacement vector in the global coordinate system are equal to:

$$d_X = -2\,\text{cm}\cdot\cos\left(40^{\circ}\right) = -1.5321\,\text{cm},$$

$$d_Y = -2\,\text{cm}\cdot\sin\left(40^{\circ}\right) = -1.2856\,\text{cm}.$$

As in the previous examples determine nodal displacements and internal forces in the elements of the structure.

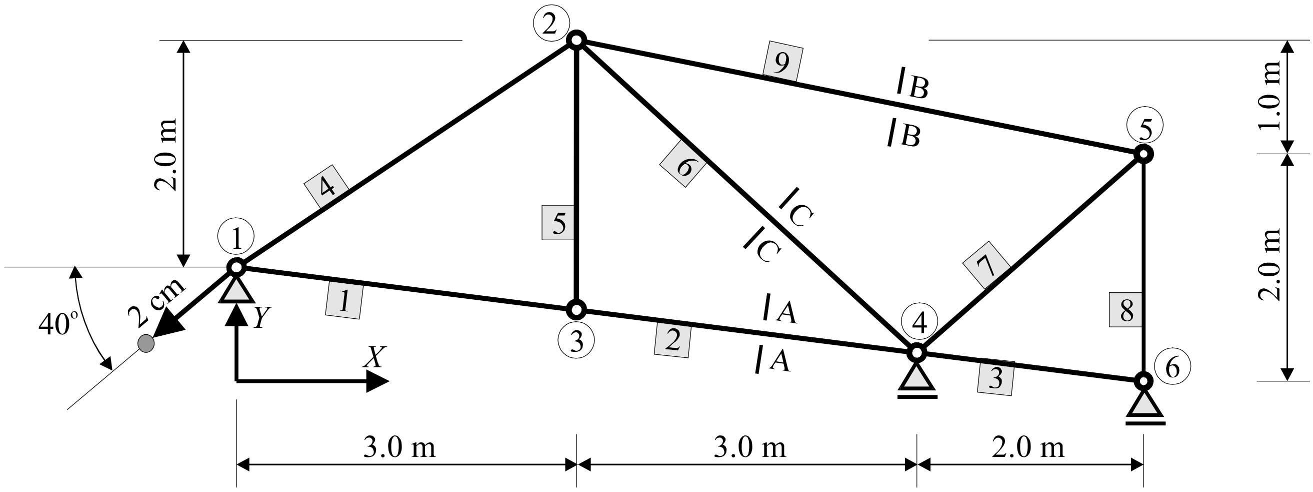

Fig.2.E4.1

The solution of the example

With regard to the geometric similarity of the structures the stiffness matrix **K** is identical to the matrix which we have obtained in example 2.E2. That is why we will not determine components of this matrix here but we will use the matrix **K** from example 2.E2. Since there are no external forces loading the truss nodes, then the nodal force vector **p** has all components equal to zero.

Boundary conditions which we have to take into consideration in this example are following:

$u_{1X} = d_X$, $u_{1Y} = d_Y$, $u_{4Y} = 0$, $u_{6Y} = 0$.

Non-zero components of the displacements of node No 1 cause some trouble with symmetrization of the stiffness matrix $\mathbf{K}^r$ because zeros cannot be inserted into columns of the matrix $\mathbf{K}$ without any consequences as we have done it in the previous examples (comp. section 2.10). We denote the vector which is the first column of the matrix $\mathbf{K}$, as $\mathbf{k}_1$ and the second column of this matrix as $\mathbf{k}_2$. Applying equation (2.73), we calculate the nodal force vector caused by the displacements of node No 1

$$\mathbf{p}^d = \mathbf{p} - d_X \mathbf{k}_1 - d_Y \mathbf{k}_2$$

$$\mathbf{p}^d(d_X) = 1.532\text{E-2}\cdot \left[\begin{array}{c} 18986.7 \\ 4923.1 \\ \hline -9216.6 \\ -6144.4 \\ \hline -9770.1 \\ 1221.3 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \end{array}\right] \begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array} \qquad \mathbf{p}^d(d_Y) = 1.286\text{E-2}\cdot \left[\begin{array}{c} 4923.1 \\ 4248.9 \\ \hline -6144.4 \\ -4096.2 \\ \hline 1221.3 \\ -152.7 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \end{array}\right] \begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

$$\mathbf{p}^d = \left[\begin{array}{c} 354.1866 \\ 130.0507 \\ \hline -220.2000 \\ -146.7990 \\ \hline -133.9870 \\ 16.7484 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \end{array}\right] \begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

Now we insert the values of displacements of node No 1 and other boundary conditions into the obtained vector. The modified vector

$$\mathbf{p}^{dr} = \left[\begin{array}{c} \mathit{-0.01532} \\ \mathit{-0.01286} \\ \hline -220.2000 \\ -146.7990 \\ \hline -133.9870 \\ 16.7484 \\ \hline 0.0 \\ \mathit{0} \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ \mathit{0} \end{array}\right] \begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array},$$

will be applied as „the right side” of the set of equations. After these operations we can take into consideration the boundary conditions in the matrix **K** in a standard way and we obtain the same matrix $\mathbf{K}^r$ as in example 2.E2. In the matrices $\mathbf{p}^{dr}$ and $\mathbf{K}^r$ the modified components are marked in italic fonts.

We solve the set of equations (2.74) obtaining the nodal displacement vector **u** and we calculate the reaction vector of supports $\mathbf{r} = \mathbf{K}\,\mathbf{u} - \mathbf{p}^d$:

$$\mathbf{u}=\left[\begin{array}{c} -1.532E-2 \\ -1.286E-2 \\ \hline -1.88116E-2 \\ -0.72301E-2 \\ \hline -1.48631E-2 \\ -0.72301E-2 \\ \hline -1.42061E-2 \\ 0.0 \\ \hline -1.67293E-2 \\ 2.38613E-4 \\ \hline -1.42061E-2 \\ 0.0 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array} \qquad \mathbf{r}=\left[\begin{array}{c} 0.0 \\ -1.909 \\ ------ \\ ------ \\ ------ \\ ------ \\ ------ \\ 7.636 \\ ------ \\ ------ \\ ------ \\ -5.727 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

The scheme of element displacements of the truss is presented in Fig.2.E4.2.

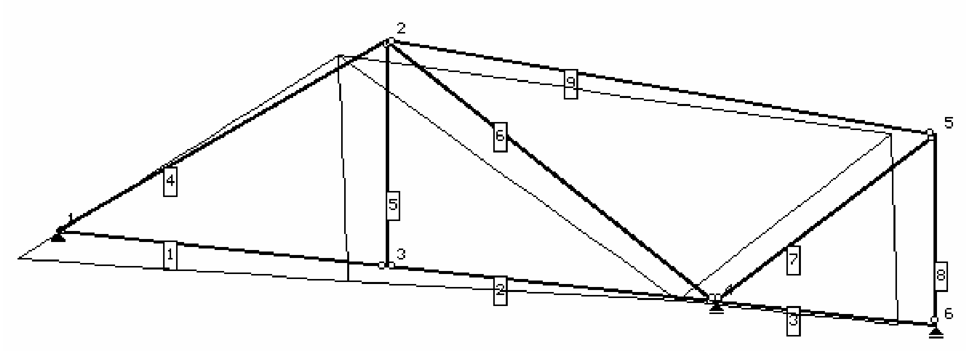

Fig.2.E4.2

Tab.2.E4.1 contains nodal displacements, internal forces and stresses in the elements of the truss calculated according to equations (2.76) and (2.77).

Tab.2.E4.1

We obtain columns marked by u_{iX}, u_{iY}, u_{jX}, u_{jY} in the way shown in Fig.2.14

No n	u_{iX} [m]	u_{iY} [m]	u_{jX} [m]	u_{jY} [m]	u_{ix} [m] (2.22)	u_{jx} [m] (2.22)	N [kN] (2.76)	σ [kN/m^2] (2.77)
1	-1.5320E-2	-1.2860E-2	-1.48631E-2	-0.72301E-2	-1.36066E-2	-1.38515E-2	-2.430	-972
2	-1.48631E-2	-0.72301E-2	-1.42061E-2	0.0	-1.38515E-2	-1.40964E-2	-2.430	-972
3	-1.42061E-2	0.0	-1.42061E-2	0.0	-1.40964E-2	-1.40964E-2	0.0	0
4	-1.532E-2	-1.286E-2	-1.88116E-2	-0.72301E-2	-1.98805E-2	-1.96627E-2	2.898	725
5	-1.48631E-2	-0.72301E-2	-1.88116E-2	-0.72301E-2	-7.23010E-3	-7.23010E-3	0.0	0
6	-1.88116E-2	-0.72301E-2	-1.42061E-2	0.0	-8.98148E-3	-1.04721E-2	-3.956	-4395
7	-1.42061E-2	0.0	-1.67293E-2	2.38613E-4	-1.06912E-2	-1.24329E-2	-7.078	-7865
8	-1.42061E-2	0.0	-1.67293E-2	2.38613E-4	0.0	2.38613E-4	5.727	1432
9	-1.88116E-2	-0.72301E-2	-1.67293E-2	2.38613E-4	-1.70284E-2	-1.64512E-2	5.433	1358