Example 2.E3.

The content of the example

The truss with the geometry and load identical to those used in previous example 2.E1, differs from it in the kind of support applied at node No 6. This is a movable support with the displacement which is not parallel to any axes of the global coordinate system.

Determine nodal displacements, internal forces and stresses in the elements of the structure.

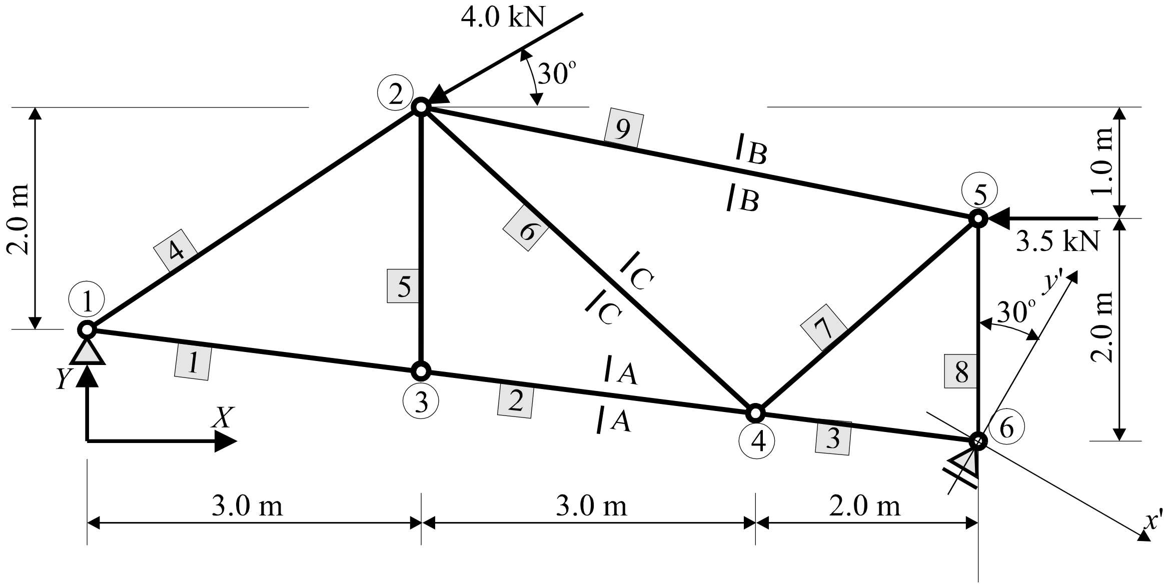

Fig.2.E3.1

The solution of the example

The solution of the example (we calculate nodal displacements, internal forces and stresses in the elements) will require different formulation of the stiffness matrix **K** because the set of equations in which it occurs has to contain equilibrium equations for node No 6 expressed with regard to the local coordinate system *x'y'*(referring to the support, comp. Chapter II section 2.7).

The nodal forces vector $\mathbf{p}$ and $\mathbf{p}^r$ are identical to those presented in example 2.E1. The matrices $\mathbf{J}^e$ (blocks of the stiffness matrix) are also exactly the same.

The stiffness matrices of elements touching the support should be transformed so that forces in node No 6 are expressed in the local support coordinate system *x'y'*. The transformation matrix for this coordinate system (comp. equation. (2.55)) is equal to

$$\mathbf{R'}_6 = \begin{bmatrix} \cos(-30^o) & -\sin(-30^o) \\ \sin(-30^o) & \cos(-30^o) \end{bmatrix} = \begin{bmatrix} 0.866025 & 0.5 \\ -0.5 & 0.866025 \end{bmatrix}$$

The element transformation matrices are equal to (comp. (2.61) and (2.62)):

- transformation matrix for element No 3

$$\mathbf{R'}^3 = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{R'}_6 \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0.866025 & 0.5 \\ 0 & 0 & -0.5 & 0.866025 \end{array}\right]$$

- transformation matrix for element No 8

$$\mathbf{R'}^8 \begin{bmatrix} \mathbf{R'}_6 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = \left[\begin{array}{cc|cc} 0.866025 & 0.5 & 0 & 0 \\ -0.5 & 0.866025 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

The transformation matrices for elements No 3 and 8 are obtained on the basis of equation (2.60)

$$\mathbf{K'}^3 = \left(\mathbf{R'}^3\right)^T \mathbf{K}^3 \mathbf{R'}^3; \; \mathbf{K'}^8 = \left(\mathbf{R'}^8\right)^T \mathbf{K}^8 \mathbf{R'}^8$$

$$\mathbf{K'}^3 = \left[\begin{array}{cc|cc} 14655.18 & -1831.9 & -13607.7 & -5741.12 \\ -1831.9 & 228.99 & 1700.97 & 717.64 \\ \hline -13607.7 & 1700.97 & 12635.09 & 5330.77 \\ -5741.12 & 717.64 & 5330.77 & 2249.07 \end{array}\right] = \begin{bmatrix} \mathbf{J}^3 & -\mathbf{J'}^3 \\ -\left(\mathbf{J'}^3\right)^T & \mathbf{J''}^3 \end{bmatrix}$$

$$\mathbf{K'}^8 = \left[\begin{array}{cc|cc} 6000 & -10392.3 & 0.0 & 12000 \\ -10392.3 & 17999.98 & 0.0 & -20784.6 \\ \hline 0.0 & 0.0 & 0.0 & 0.0 \\ 12000 & -20784.6 & 0.0 & 24000 \end{array}\right] = \begin{bmatrix} \mathbf{J''}^8 & -\mathbf{J'}^8 \\ -\left(\mathbf{J'}^8\right)^T & \mathbf{J}^8 \end{bmatrix}$$

After aggregation we obtain the global stiffness matrix

$$\mathbf{K}=\begin{array}{cc}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} & \\
\left[\begin{array}{c|c|c|c|c|c}
\mathbf{J}^1+\mathbf{J}^4 & -\mathbf{J}^4 & -\mathbf{J}^1 & & & \\ \hline
-\mathbf{J}^4 & \mathbf{J}^4+\mathbf{J}^5+\mathbf{J}^6+\mathbf{J}^9 & -\mathbf{J}^5 & -\mathbf{J}^6 & -\mathbf{J}^9 & \\ \hline
-\mathbf{J}^1 & -\mathbf{J}^5 & \mathbf{J}^1+\mathbf{J}^2+\mathbf{J}^5 & -\mathbf{J}^2 & & \\ \hline
 & -\mathbf{J}^6 & -\mathbf{J}^2 & \mathbf{J}^2+\mathbf{J}^3+\mathbf{J}^6+\mathbf{J}^7 & -\mathbf{J}^7 & -\mathbf{J}'^3 \\ \hline
 & -\mathbf{J}^9 & & -\mathbf{J}^7 & \mathbf{J}^7+\mathbf{J}^8+\mathbf{J}^9 & -\mathbf{J}'^8 \\ \hline
 & & & -\left(\mathbf{J}'^3\right)^{\mathrm{T}} & -\left(\mathbf{J}'^8\right)^{\mathrm{T}} & \mathbf{J}''^3+\mathbf{J}''^8
\end{array}\right] & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\end{array}$$

and after inserting the suitable values, we obtain the global stiffness matrix **K** in the form presented on the next page.

Boundary conditions are written by equations:

$u_{1X} = 0$, $u_{1Y} = 0$, $u_{4Y} = 0$, $u_{6y} = 0$.

Since equilibrium equations for node No 6 are written with the use of components of force vectors in the local system *x'y'*, then the way of considering boundary conditions is identical to that occurring in example 2.E1. In order to obtain the matrix $\mathbf{K}^r$ (presented on the successive page after the stiffness matrix **K**) the rows of the matrix **K** numbered 1, 2 and 12 should be modified.

The global stiffness matrix:

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \quad\quad 1 \quad\quad & \quad\quad 2 \quad\quad & \quad\quad 3 \quad\quad & \quad\quad 4 \quad\quad & \quad\quad 5 \quad\quad & \quad\quad 6 \quad\quad \end{array} \\
\mathbf{K}= & \left[\begin{array}{cc|cc|cc|cc|cc|cc}
18986.7 & 4923.1 & -9216.6 & -6144.4 & -9770.1 & 1221.3 & 0 & 0 & 0 & 0 & 0 & 0 \\
4923.1 & 4248.9 & -6144.4 & -4096.2 & 1221.3 & -152.7 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
-9216.6 & -6144.4 & 19710.1 & 3012.2 & 0 & 0 & -1442.0 & 1321.9 & -9051.5 & 1810.3 & 0 & 0 \\
-6144.4 & -4096.2 & 3012.2 & 10217.4 & 0 & -4547.4 & 1321.9 & -1211.7 & 1810.3 & -362.1 & 0 & 0 \\
\hline
-9770.1 & 1221.3 & 0 & 0 & 19540.2 & -2442.5 & -9770.1 & 1221.3 & 0 & 0 & 0 & 0 \\
1221.3 & -152.7 & 0 & -4547.4 & -2442.5 & 4852.7 & 1221.3 & -152.7 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & -1442.0 & 1321.9 & -9770.1 & 1221.3 & 28169.0 & -2361.1 & -2301.7 & -2014.0 & -13607.7 & -5741.1 \\
0 & 0 & 1321.9 & -1211.7 & 1221.3 & -152.7 & -2361.1 & 3355.6 & -2014.0 & -1762.2 & 1701.0 & 717.6 \\
\hline
0 & 0 & -9051.5 & 1810.3 & 0 & 0 & -2301.7 & -2014.0 & 11353.2 & 203.7 & 0 & 12000.0 \\
0 & 0 & 1810.3 & -362.1 & 0 & 0 & -2014.0 & -1762.2 & 203.7 & 26124.3 & 0 & -20784.6 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & -13607.7 & 1701.0 & 0 & 0 & 18635.1 & -5061.5 \\
0 & 0 & 0 & 0 & 0 & 0 & -5741.1 & 717.6 & 12000.0 & -20784.6 & -5061.5 & 20249.1
\end{array}\right]
\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}
\end{array}
$$

The global stiffness matrix after taking into consideration boundary conditions:

$$
\begin{array}{ccc}
 & \begin{array}{cccccccccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} & \multicolumn{2}{c}{4} & \multicolumn{2}{c}{5} & \multicolumn{2}{c}{6} \end{array} & \\
\mathbf{K}^{r} = & \left[\begin{array}{cc|cc|cc|cc|cc|cc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 19710.1 & 3012.2 & 0 & 0 & -1442.0 & 1321.9 & -9051.5 & 1810.3 & 0 & 0 \\
0 & 0 & 3012.2 & 10217.4 & 0 & -4547.4 & 1321.9 & -1211.7 & 1810.3 & -362.1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 19540.2 & -2442.5 & -9770.1 & 1221.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -4547.4 & -2442.5 & 4852.7 & 1221.3 & -152.7 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & -1442.0 & 1321.9 & -9770.1 & 1221.3 & 28169.0 & -2361.1 & -2301.7 & -2014.0 & -13607.7 & 0 \\
0 & 0 & 1321.9 & -1211.7 & 1221.3 & -152.7 & -2361.1 & 3355.6 & -2014.0 & -1762.2 & 1701.0 & 0 \\
\hline
0 & 0 & -9051.5 & 1810.3 & 0 & 0 & -2301.7 & -2014.0 & 11353.2 & 203.7 & 0 & 0 \\
0 & 0 & 1810.3 & -362.1 & 0 & 0 & -2014.0 & -1762.2 & 203.7 & 26124.3 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & -13607.7 & 1701.0 & 0 & 0 & 18635.1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] & \begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}
\end{array}
$$

After solving the set of equations (2.52), we obtain the nodal displacement vector **u** and next on the basis of equation (2.75) we obtain the vector **r**:

$$\mathbf{u}=\left[\begin{array}{c} 0.0 \\ 0.0 \\ \hline -9.86061\text{ E-4} \\ 7.75179\text{ E-4} \\ \hline -2.03717\text{E-4} \\ 7.75179\text{E-4} \\ \hline -5.89545\text{E-4} \\ 0.93464\text{E-4} \\ \hline -13.2740\text{E-4} \\ 3.57736\text{E-4} \\ \hline -6.69398\text{E-4} \\ 0.0 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array} \qquad \mathbf{r}=\left[\begin{array}{c} 7.262 \\ 2.516 \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ \text{------} \\ \hline \text{------} \\ -0.596 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

The displacement u_{6x} (row No 11 of the vector **u**) is the component parallel to the x' axis and other components of the vector **u** are expressed with regard to the axes of the global coordinate system XY. The reaction V_{6y} (row No 12 of the vector **r**) is the force parallel to the y' axis of the local support system.

The scheme of element displacement of the truss is presented in Fig.2.E3.2.

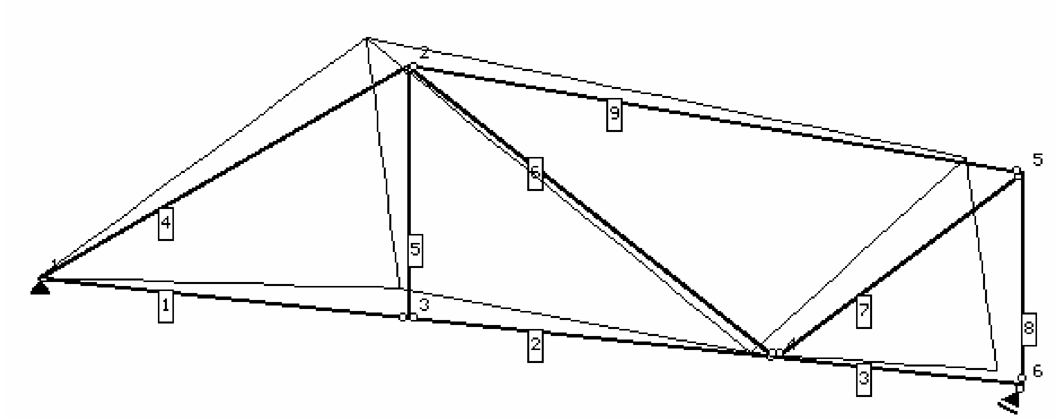

Fig.2.E3.2

Tab.2.E3.1 presents nodal displacements, internal forces and stresses in the elements of the structure. Given in the table displacements for node No 6 were earlier transformed to the global system according to equation (2.55).

Tab.2.E3.1

We obtain the columns marked by u_{iX}, u_{iY}, u_{jX}, u_{jY} in the way shown in Fig.2.14

No n	u_{iX} [m]	u_{iY} [m]	u_{jX} [m]	u_{jY} [m]	u_{ix} [m] (2.22)	u_{jx} [m] (2.22)	N [kN] (2.75)	σ [kN/m^2] (2.76)
1	0.0	0.0	-2.03717E-4	7.75179E-4	0.0	-2.98293E-4	-2.960	-1184.0
2	-2.03717E-4	7.75179E-4	-5.89545E-4	0.93464E-4	-2.98293E-4	-5.96585E-4	-2.960	-1184.0
3	-5.89545E-4	0.93464E-4	-5.79731E-4	3.34672E-4	-5.96587E-4	-6.16767E-4	-0.300	-120.1
4	0.0	0.0	-9.86061E-4	7.75179E-4	0.0	-3.90461E-4	-5.198	-1299.5
5	-2.03717E-4	7.75179E-4	-9.86061E-4	7.75179E-4	7.75179E-4	7.75179E-4	0.000	0.0
6	-9.86061E-4	7.75179E-4	-5.89545E-4	0.93464E-4	-1.25068E-3	-4.97741E-4	1.998	2220.1
7	-5.89545E-4	0.93464E-4	-13.2740E-4	3.57736E-4	-3.82131E-4	-7.63398E-4	-1.549	-1721.6
8	-5.79731E-4	3.34672E-4	-13.2740E-4	3.57736E-4	3.34672E-4	3.57736E-4	0.554	138.4
9	-9.86061E-4	7.75179E-4	-13.2740E-4	3.57736E-4	-1.11894E-3	-1.37178E-3	-2.380	-595.0