Example 2.E2.

The content of the example

The truss with geometry similar to that presented in the previous example undergoes the action of a temperature. Element No 3 is warmed by 30 K, element No 7 by 40 K and element No 8 by 30 K. Fig.2.E2.1 shows the scheme of the temperature load of that structure. The material of the truss is characterised by Young's modulus $E=1.2 \cdot 10^7$ kPa and by the expansion coefficient $\alpha_t=0.00001/K$.

Determine the nodal displacement of the truss, internal forces and stresses for the elements of the structure.

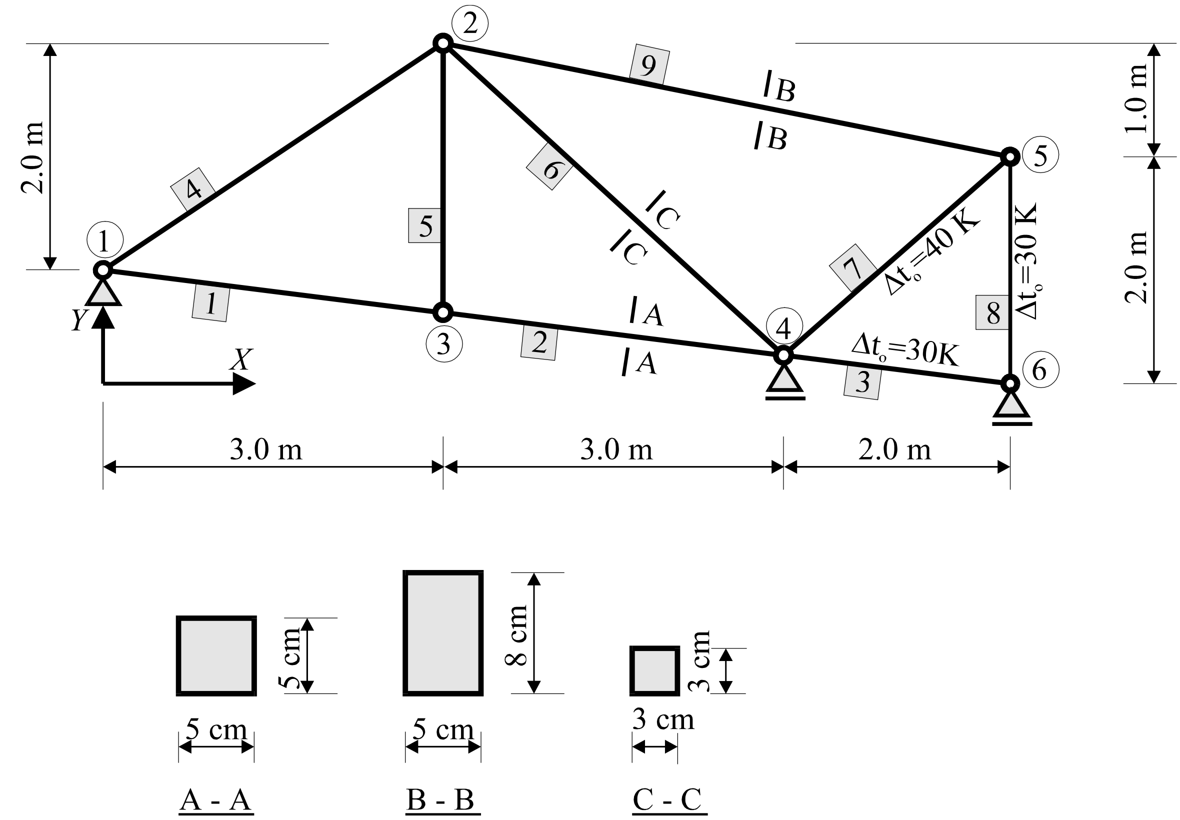

Fig.2.E2.1

The juxtaposition of nodal data is shown in Tab.2.E1.1, and the juxtaposition of element data is in Tab.2.E1.2.

The solution of the example

The truss presented in Fig.2.E2.1 is rested on three supports. Since this truss is a nondetermined static structure (once), then the temperature load causes the change of internal forces for its elements. For determined static structures (as in the previous example) the temperature load does not cause internal forces, though nodal displacements exist.

We start the example from building a nodal force vector for the elements loaded with a temperature and for this purpose we use equations (2.70) and (2.71) obtaining

$$\mathbf{f}^{3t}=\begin{bmatrix} 9.0 \\ 0.0 \\ -9.0 \\ 0.0 \end{bmatrix}\begin{matrix} i \\ \\ j \\ \\ \end{matrix}$$ - nodal force vector of element No 3 in the local system

$$\mathbf{f}^{3t}=\begin{bmatrix} 8.93052 \\ -1.11632 \\ -8.93052 \\ 1.11632 \end{bmatrix}\begin{array}{cc} \text{Nodal local number} & \text{Nodal global number} \\ i & 4 \\ & \\ j & 6 \\ & \end{array}$$ - nodal force vector of element No 3 in the global system

$$\mathbf{f}^{7t}=\begin{bmatrix} 4.32 \\ 0.0 \\ -4.32 \\ 0.0 \end{bmatrix}\begin{matrix} i \\ \\ j \\ \\ \end{matrix} \qquad \mathbf{f}^{7t}=\begin{bmatrix} 3.25113 \\ 2.84474 \\ -3.25113 \\ -2.84474 \end{bmatrix}\begin{matrix} i & 4 \\ & \\ j & 5 \\ & \end{matrix}$$

$$\mathbf{f}^{8t}=\begin{bmatrix} 14.4 \\ 0.0 \\ -14.4 \\ 0.0 \end{bmatrix}\begin{matrix} i \\ \\ j \\ \\ \end{matrix} \qquad \mathbf{f}^{8t}=\begin{bmatrix} 14.4 \\ 0.0 \\ -14.4 \\ 0.0 \end{bmatrix}\begin{matrix} i & 6 \\ & \\ j & 5 \\ & \end{matrix}$$

After aggregation depending on the summation of forces in nodes No 4, 5 and 6, we obtain the vector of the nodal force caused by the temperature load:

$$\mathbf{p}=\left[\begin{array}{c} 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline -12.18165 \\ -1.72842 \\ \hline 3.25113 \\ 17.24474 \\ \hline 8.93052 \\ -15.51632 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

Boundary conditions are written by equations: $u_{1X} = 0$, $u_{1Y} = 0$, $u_{4Y} = 0$ and $u_{6Y} = 0$. Global numbers of degrees of freedom are following: $u_{1X} \rightarrow$ No 1; $u_{1Y} \rightarrow$ No 2; $u_{4Y} \rightarrow$ No 8; $u_{6Y} \rightarrow$ No 12. Hence the consideration of the boundary conditions in the vector **p** depends on the substitution of zeros for the values in rows 1, 2, 8, 12 of this vector. After this operation, we obtain the vector $\mathbf{p}^r$ which is applied as the *right side* vector of the set of equations:

$$\mathbf{p}^r=\left[\begin{array}{c} \mathit{0} \\ \mathit{0} \\ \hline 0.0 \\ 0.0 \\ \hline 0.0 \\ 0.0 \\ \hline -12.18165 \\ \mathit{0} \\ \hline 3.25113 \\ 17.24474 \\ \hline 8.93052 \\ \mathit{0} \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

We obtain the stiffness matrix **K** in the way similar to the one used in the previous example. With regard to the geometric similarity of the structure this matrix is identical to the matrix **K** from example 2.E1.

The boundary conditions require the modification of four rows of the matrix **K** and obviously these are rows numbered 1, 2, 8 and 12 which have been changed in the vector **p**. After symmetrizing the matrix **K**, we obtain the matrix $\mathbf{K}^r$ (located on the next page) of which modified components are written in italic fonts.

After solving the set of equations

$$\mathbf{K}^r\,\mathbf{u}=\mathbf{p}^r\,,$$

we obtain the nodal displacement vector **u** and the constraint reaction vector **r** calculated on the basis of relation (2.75):

$$\mathbf{u}=\left[\begin{array}{c} 0.0 \\ 0.0 \\ \hline 1.32415\text{E-4} \\ -1.33262\text{E-4} \\ \hline -5.77624\text{E-5} \\ -1.33262\text{E-4} \\ \hline -8.22092\text{E-5} \\ 0.0 \\ \hline 3.85038\text{E-4} \\ 6.39741\text{E-4} \\ \hline 5.27167\text{E-4} \\ 0.0 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array} \qquad \mathbf{r}=\left[\begin{array}{c} 0.0 \\ -0.31793 \\ \text{------} \\ \text{------} \\ \text{------} \\ \text{------} \\ \text{------} \\ 1.27172 \\ \text{------} \\ \text{------} \\ \text{------} \\ -0.95379 \end{array}\right]\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}$$

The global stiffness matrix after taking into consideration boundary conditions:

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} & \multicolumn{2}{c}{4} & \multicolumn{2}{c}{5} & \multicolumn{2}{c}{6} \end{array} \\
\mathbf{K}^{r} = & \left[\begin{array}{cc|cc|cc|cc|cc|cc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 19710.1 & 3012.2 & 0 & 0 & -1442.0 & 0 & -9051.5 & 1810.3 & 0 & 0 \\
0 & 0 & 3012.2 & 10217.4 & 0 & -4547.4 & 1321.9 & 0 & 1810.3 & -362.1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 19540.2 & -2442.5 & -9770.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -4547.4 & -2442.5 & 4852.7 & 1221.3 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & -1442.0 & 1321.9 & -9770.1 & 1221.3 & 28169.0 & 0 & -2301.7 & -2014.0 & -14655.2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & -9051.5 & 1810.3 & 0 & 0 & -2301.7 & 0 & 11353.2 & 203.7 & 0 & 0 \\
0 & 0 & 1810.3 & -362.1 & 0 & 0 & -2014.0 & 0 & 203.7 & 26124.3 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & -14655.2 & 0 & 0 & 0 & 14655.2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\begin{array}{c} 1 \\ \\ 2 \\ \\ 3 \\ \\ 4 \\ \\ 5 \\ \\ 6 \\ \\ \end{array}
\end{array}
$$

The nodal displacements of the structure are shown in Fig.2.E2.2.

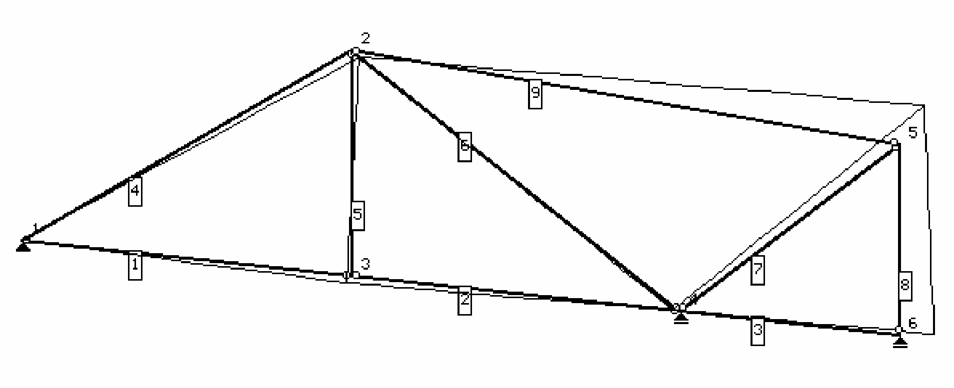

Fig.2.E2.2

Tab.2.E2.1 presents the values of nodal displacements, internal forces and stresses for the elements of the structure calculated according to equations (2.76) and (2.79).

Tab.2.E2.1

We obtain columns marked by u_{iX}, u_{iY}, u_{jX}, u_{jY} in the way shown in Fig.2.14

No n	u_{iX} [m]	u_{iY} [m]	u_{jX} [m]	u_{jY} [m]	u_{ix} [m] (2.22)	u_{jx} [m] (2.22)	Nu [kN] (2.76)	Nt [kN]	$N=Nt+Nu$ [kN]	σ [N/m^2] (2.79)
1	0.0	0.0	-5.77624E-5	-1.33262E-4	0.0	-4.07872E-4	-0.405	0.0	-0.405	-161.9
2	-5.77624E-5	-1.33262E-4	-8.22092E-5	0.0	-4.07872E-4	-8.15743E-4	-0.405	0.0	-0.405	-161.9
3	-8.22092E-5	0.0	5.27167E-4	0.0	-8.15746E-4	5.23097E-3	9.0	-9.0	0.000	0.0
4	0.0	0.0	1.32415E-4	-1.33262E-4	0.0	3.62555E-4	0.483	0.0	0.483	120.7
5	-5.77624E-5	-1.33262E-4	1.32415E-4	-1.33262E-4	-1.33262E-3	-1.33262E-3	0.0	0.0	0.0	0.0
6	1.32415E-4	-1.33262E-4	-8.22092E-5	0.0	1.87658E-3	-6.06008E-4	-0.659	0.0	-0.659	-732.0
7	-8.22092E-5	0.0	3.85038E-4	6.39741E-4	-6.18686E-4	7.11042E-3	3.141	-4.320	-1.179	-1310.0
8	5.27167E-4	0.0	3.85038E-4	6.39741E-4	0.0	6.39741E-3	15.354	-14.4	0.954	238.4
9	1.32415E-4	-1.33262E-4	3.85038E-4	6.39741E-4	1.55978E-3	2.52097E-3	0.905	0.0	0.905	226.2