THE FINITE ELEMENT METHOD IN STATIC

PROBLEMS FOR ENGINEERING STRUCTURES

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INTRODUCTION

This book deals with the use of the finite element method (FEM is an abbreviation for the Finite Elements Method or FEA for the Finite Elements Analysis which is another name for FEM) to solve linear problems of solid mechanics. We are particularly interested in statics of bar structures (trusses, frames), surface girders (two-dimensional plates, three-dimensional plates, shells); elements that are very often used in engineering structures. Obviously there are many books which discuss these problems, some of them beautiful and thick, for example, the books written by the creators of FEM like O. C. Zienkiewicz [19],[20], J. H. Argyris, K. J. Bathe[1]. In our opinion there are not enough books on the Polish market that would introduce difficult FEM problems in a simple way so that the cognition of its theoretical bases would be possible for people who do not deal with structure mechanics in every day practice. The cognition of the FEM bases is necessary for a contemporary designer who has to use computer packages of programmes helpful in designing and whose calculated modules are just based on the finite element method. The book of G. Rakowski and Z. Kacprzyk [12], which can be treated as a manual, requires some preparation on the part of readers and besides it lacks simple exercises to be done by the reader without the help of a computer. A good example of a FEM manual used in the United Kingdom is the book written by C. T. F. Ross [14].

Hence we have decided to write a manual for engineers which would be as simple as possible (but without trivialising problems) in order to simplify the study and cognition of FEM by yourself. The content of this book is based on lectures which have been given by one of the co-authors (J.P.) at the Faculty of Civil and Sanitary Engineering of the Technical University of Lublin since 1990. Yet the content of this book has been much more broadened and deepened in comparison with the lectures. We have also elaborated many examples simplifying the cognition of detailed problems and algorithms of FEM.

In order to study this book the reader should have basic knowledge of akin sciences, in particular those concerning strength of materials and bases of the theory of elasticity. We assume that the reader is familiar with notions like stress, strain, constitutive relations (particularly the generalised Hook's law), thus we do not pay much attention to these problems. References given at the end of the book elaborate the mentioned topics in detail. Special attention should be paid to the books on the theory of elasticity written by Y. C. Fung

[3], S. P. Timoshenka [17] and the book on the strength of materials written by P. Jastrzêbski, J. Mutermilch and W. Or³owski [8].

The study of FEM problems requires the use of matrix algebra and we assume that the reader knows the bases of calculus. At the end of the book in Appendix 1 one can find a short review of the most important information concerning matrix algebra necessary for reading this book.

The knowledge of numerical methods may not be essential for understanding FEM because it is rather joined with computer implementation of algorithms but on the other hand the information on them helps to use ready made packages of programmes applying to FEM. Since numerical methods are not always in the programme of the university course in mathematics, we provide the review of methods of storage of stiffness matrices and of solving large sets of linear equations in Appendix 2. We also encourage the reader to be familiar with the book written by A. Georg and J. Lieu [5] because it is particularly devoted to these methods. The study of this book is also simplified by locating the list of notations and their interpretation at the beginning of it.

Moreover, included exercises concerning bar structures have been designed with the help of PRET_r2 programme (ver.3.1) which has been designed by J. Podgórski.

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We also thank all the reviewers and colleagues who have read the book before printing to find errors and improve the clarity of the material.

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NOTATION

$\mathbf{a}$, $\mathbf{b}$, $\mathbf{u}$ - column matrix - vectors

$\mathbf{A}$, $\mathbf{B}$, $\mathbf{K}$ - two-dimensional matrix

$\mathbf{u}'$, $\mathbf{K}'$, u_x - vectors, matrices and scalars in the local coordinate system of an element

$\mathbf{u}$, $\mathbf{K}$, u_X - vectors, matrices and scalars in the global coordinate system

x, *y*, *z* - axes of the local coordinate system of an element

X, Y, Z - axes of the global coordinate system

$\mathbf{q}_i$ - lower index at vectors or matrices denotes the node number *i*

$\mathbf{q}^e$ - upper index at vectors or matrices denotes the element number *e*

$u_x, u_y, u_z, \varphi_x, \varphi_y, \varphi_z$ - components of the local vector $\mathbf{u}$ in the local coordinate system

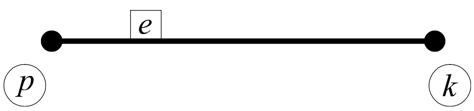
$u_X, u_Y, u_Z, \varphi_X, \varphi_Y, \varphi_Z$ - components of the local vector $\mathbf{u}$ in the global coordinate system

$\mathbf{u}_i = \begin{bmatrix} u_{iX} \\ u_{iY} \\ u_{iZ} \end{bmatrix}$ - displacement vector of node *i*

$\mathbf{f}_i = \begin{bmatrix} F_{iX} \\ F_{iY} \\ F_{iZ} \end{bmatrix}$ - force vector of node *i*

$\mathbf{u}^e = \begin{bmatrix} \mathbf{u}_p \\ \mathbf{u}_k \end{bmatrix}$ - nodal displacement vector of an element

$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$, $\mathbf{f} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$ - components of the vector are usually denoted by small letters just as a vector except for the nodal forces vector which is denoted by capital letters in accordance with the tradition.



Element numbers are situated more closely to the first node.

det ($\mathbf{A}$) stands for the determinant of the matrix $\mathbf{A}$

$\mathbf{A}^{\mathrm{T}}$ means transpose of the matrix **A** which means that if $\mathbf{B} = \mathbf{A}^{\mathrm{T}}$, then $B_{ij} = A_{ji}$

N_N - number of nodes in a structure

N_E - number of elements in a structure

N_D - number of degrees of freedom of one node

N_K - number of degrees of freedom of the whole structure

N_{De} - number of degrees of freedom of an element

E - Young's modulus (modulus of elasticity)

G - Kirchhoff's modulus (modulus of elasticity in shear)

ν - Poisson's ratio

C - torsional resistance