Obliczanie ugięcia płyty podpartej przegubowo na 2 krawędziach a na 2 sztywno zamocowana - schemat a

$ORIGIN := 1$

$E := 18GPa \qquad \nu := 0.22 \qquad h := 6cm \qquad Lx := 5m \qquad Ly := 3m$

$p0 := -7kPa$

$$D0 := \frac{E \cdot h^3}{12(1-\nu^2)} = 340.479 \cdot kN \cdot m$$ *- sztywność płytowa*

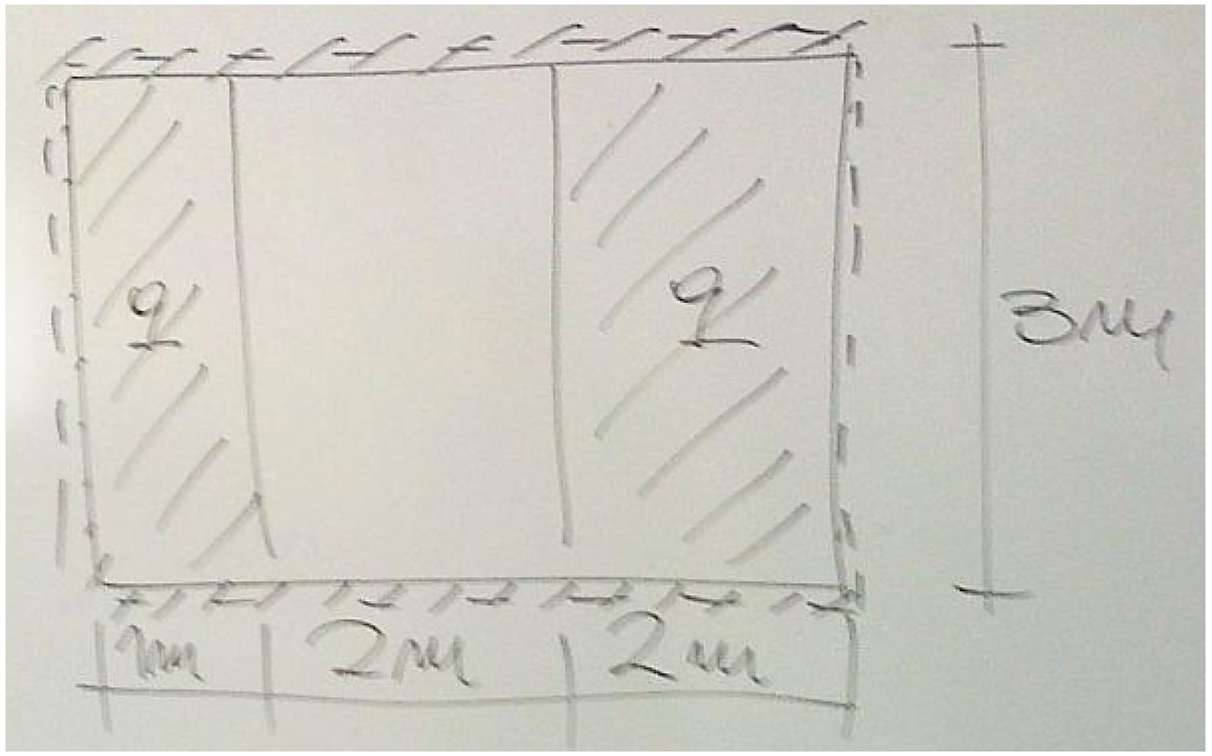
Funkcja obciążenia płyty: $q(x) := 1$

Obciążenie ciągłe p0, równomiernie rozłożone na obszarze płyty:
Lx1 < x < Lx2 , 0 < y < Ly i ciężar własny p1

$Lx1 := 0m \qquad Lx2 := 1m \qquad Lx3 := 3m \qquad Lx4 := Lx$

Q - wypadkowa obciążenia ciągłego

$$Q0 := p0 \cdot Ly \cdot \left(\int_{Lx1}^{Lx2} q(x)\, dx + \int_{Lx3}^{Lx4} q(x)\, dx \right) \qquad Q0 = -63 \cdot kN$$



Metoda Levy'ego

Rozwinięcie obciążenia w pojedynczy szereg Fouriera

$N := 15$

$i := 1 .. N$

$$\alpha_i := \frac{i \cdot \pi}{Lx} \qquad p_i := \frac{2}{Lx} \cdot \left(\int_{Lx1}^{Lx2} p0 \cdot sin(\alpha_i \cdot x)\, dx + \int_{Lx3}^{Lx4} p0 \cdot sin(\alpha_i \cdot x)\, dx \right)$$

$$E_i := \frac{p_i}{D0 \cdot (\alpha_i)^4} \qquad \lambda_i := \alpha_i \cdot \frac{Ly}{2}$$

$\alpha_i =$ [$\frac{1}{m}$]

0.628319
1.256637
1.884956
2.513274
3.141593
3.769911
4.398230
5.026548
5.654867
6.283185
6.911504
7.539822
...

$p_i =$ [$\cdot kPa$]

	1
1	-3.930
2	2.491
3	-4.632
4	-1.246
5	-1.783
6	-0.830
7	-1.985
8	0.623
9	-0.437
10	0.000
11	-0.357
12	0.415
13	...

$E_i =$ [$\cdot mm$]

	1
1	-74.066183
2	2.934088
3	-1.077563
4	-0.091690
5	-0.053746
6	-0.012074
7	-0.015580
8	0.002865
9	-0.001254
10	0.000000
11	-0.000460
12	0.000377
13	...

$\lambda_i =$

	1
1	0.942
2	1.885
3	2.827
4	3.770
5	4.712
6	5.655
7	6.597
8	7.540
9	8.482
10	9.425
11	10.367
12	11.310
13	...

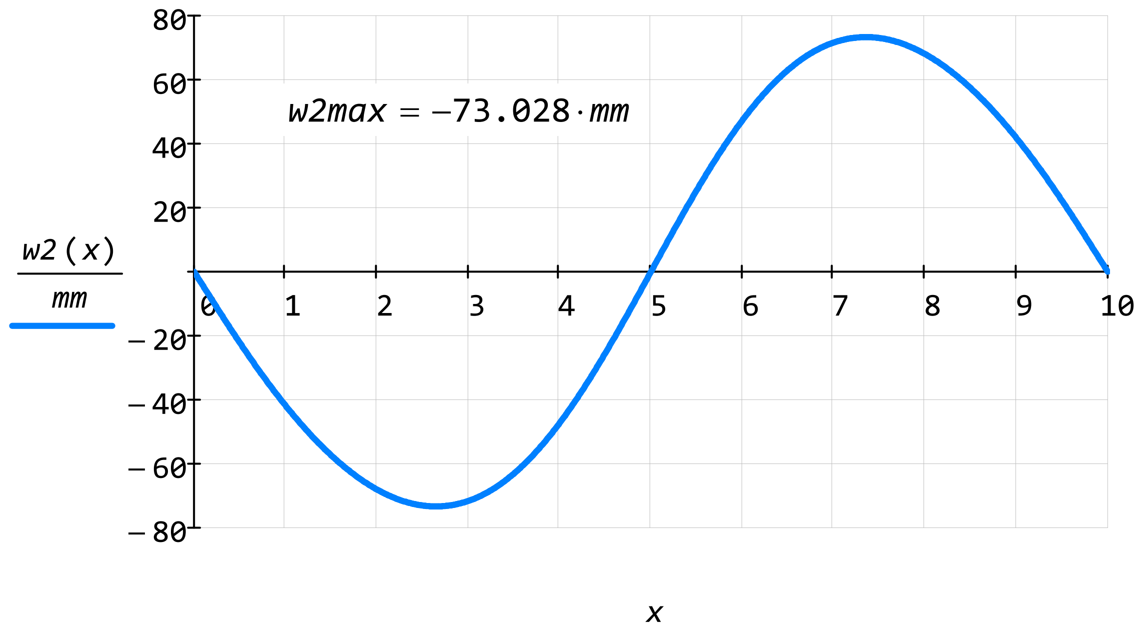
Obciążenie przybliżone szeregiem Fouriera

$$p1(x) := \sum_{i} \left(p_i \cdot sin\left(\alpha_i \cdot x\right)\right)$$

Walcowe ugięcie płyty przybliżone szeregiem Fouriera

$$w2(x) := \sum_{i} \left(E_i \cdot sin\left(\alpha_i \cdot x\right)\right)$$

$w2max := w2(2.5m)$



$w2max = -73.028 \cdot mm$

Funkcja ugięcia płyty przybliżona szeregiem Fouriera

$$C_i = \frac{E_i}{\mathrm{ch}\,\lambda_i + \dfrac{\lambda_i}{\mathrm{sh}\,\lambda_i}} \quad ,$$

$$B_i = -C_i\left(1 + \lambda_i\,\mathrm{cth}\,\lambda_i\right)$$

$A_i := 0$ $\qquad D_i := 0$ $\qquad C_i := \dfrac{E_i}{\lambda_i \cdot csch(\lambda_i) + cosh(\lambda_i)}$ $\qquad B_i := -C_i \cdot \left(1 + \lambda_i \cdot coth(\lambda_i)\right)$

$A_i =$ $\cdot mm$	$B_i =$ $\cdot mm$	$C_i =$ $\cdot mm$	$D_i =$ $\cdot mm$
$0\cdot10^{0}$	$7.204196\cdot10^{1}$	$-3.15985\cdot10^{1}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$-2.206331\cdot10^{0}$	$7.418935\cdot10^{-1}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$4.702299\cdot10^{-1}$	$-1.222234\cdot10^{-1}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$2.001147\cdot10^{-2}$	$-4.191832\cdot10^{-3}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$5.508001\cdot10^{-3}$	$-9.640918\cdot10^{-4}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$5.623959\cdot10^{-4}$	$-8.450721\cdot10^{-5}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$3.228788\cdot10^{-4}$	$-4.249876\cdot10^{-5}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$-2.601029\cdot10^{-5}$	$3.045763\cdot10^{-6}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$4.926453\cdot10^{-6}$	$-5.195419\cdot10^{-7}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$0\cdot10^{0}$	$0\cdot10^{0}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$3.287761\cdot10^{-7}$	$-2.892308\cdot10^{-8}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$-1.138258\cdot10^{-7}$	$9.246812\cdot10^{-9}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$8.924477\cdot10^{-8}$	$-6.73433\cdot10^{-9}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$9.220578\cdot10^{-9}$	$-6.495794\cdot10^{-10}$	$0\cdot10^{0}$
$0\cdot10^{0}$	$4.854258\cdot10^{-9}$	$-3.206847\cdot10^{-10}$	$0\cdot10^{0}$

$$f(i,y) := A_i \cdot \sinh(\alpha_i \cdot y) + B_i \cdot \cosh(\alpha_i \cdot y) + C_i \cdot \alpha_i \cdot y \cdot \sinh(\alpha_i \cdot y) + D_i \cdot \alpha_i \cdot y \cdot \cosh(\alpha_i \cdot y)$$

$$f0(i,y) := f(i,y) + E_i$$

$$f1(i,y) = \frac{d}{dy} f(i,y)$$

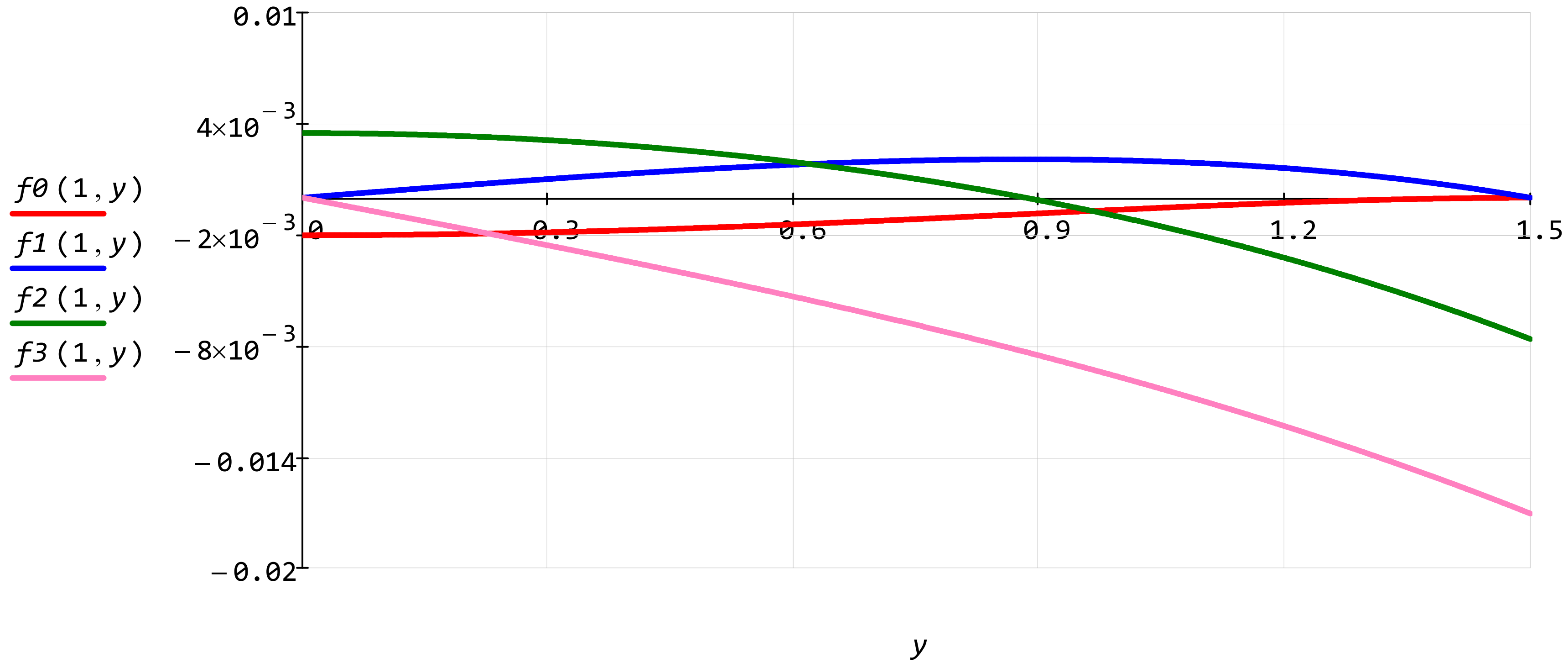
$$f1(i,y) := \alpha_i \cdot \left[A_i \cdot \cosh(\alpha_i \cdot y) + B_i \cdot \sinh(\alpha_i \cdot y) + C_i \cdot \left(\sinh(\alpha_i \cdot y) + \alpha_i \cdot y \cdot \cosh(\alpha_i \cdot y)\right) + D_i \cdot \left(\cosh(\alpha_i \cdot y) + \alpha_i \cdot y \cdot \sinh(\alpha_i \cdot y)\right)\right]$$

$$f2(i,y) = \frac{d^2}{dy^2} f(i,y)$$

$$f2(i,y) := (\alpha_i)^2 \cdot \left[A_i \cdot \sinh(\alpha_i \cdot y) + B_i \cdot \cosh(\alpha_i \cdot y) + C_i \cdot \left(2 \cdot \cosh(\alpha_i \cdot y) + \alpha_i \cdot y \cdot \sinh(\alpha_i \cdot y)\right) + D_i \cdot \left(2 \cdot \sinh(\alpha_i \cdot y) + \alpha_i \cdot y \cdot \cosh(\alpha_i \cdot y)\right)\right]$$

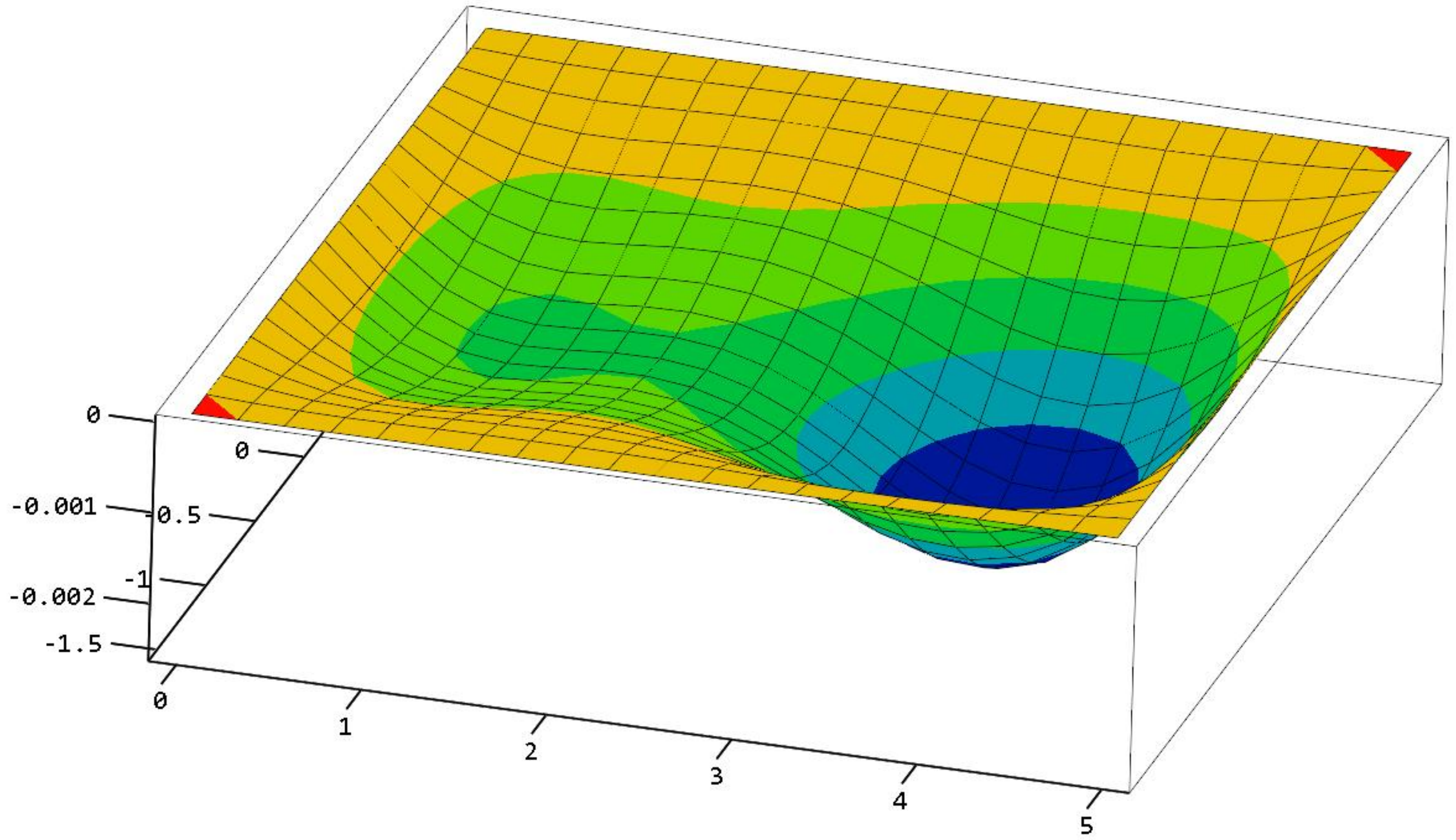
$$f3(i,y) = \frac{d^3}{dy^3} f(i,y)$$

$$f3(i,y) := (\alpha_i)^3 \cdot \left[A_i \cdot \cosh(\alpha_i \cdot y) + B_i \cdot \sinh(\alpha_i \cdot y) + C_i \cdot \left(3 \cdot \sinh(\alpha_i \cdot y) + \alpha_i \cdot y \cdot \cosh(\alpha_i \cdot y)\right) + D_i \cdot \left(3 \cdot \cosh(\alpha_i \cdot y) + \alpha_i \cdot y \cdot \sinh(\alpha_i \cdot y)\right)\right]$$



Dwa sposoby definicji funkcji ugięcia: w(x,y)=w1(x,y)

$w(x,y) := \sum_i \left(f0(i,y)\cdot\sin(\alpha_i\cdot x)\right)$ $w1(x,y) := f0(1,y)\cdot\sin(\alpha_1\cdot x)$



w

$w\left(\frac{Lx}{2}, 0\right) = -1.451\cdot mm$ $w1\left(\frac{Lx}{2}, 0\right) = -2.024\cdot mm$

Poszukiwanie miejsca ekstremalnego ugięcia

$xm := \frac{Lx}{5}$ $ym := 0$

Given

$0 < xm < Lx$ $0 < ym < Ly$

$r1 := Minimize(w, xm, ym)$

$r1 =$

	1
1	1.02787
2	0.00000

m

Ekstremalne ugięcie w1

$w(r1_1, r1_2) = -1.099\cdot mm$

$xm := \frac{Lx}{2}$ $ym := 0$

Given

$0 < xm < Lx$ $0 < ym < Ly$

$r2 := Minimize(w, xm, ym)$

$r2 =$

	1
1	3.67108
2	0.00000

m

Ekstremalne ugięcie w2

$w(r2_1, r2_2) = -2.565\cdot mm$

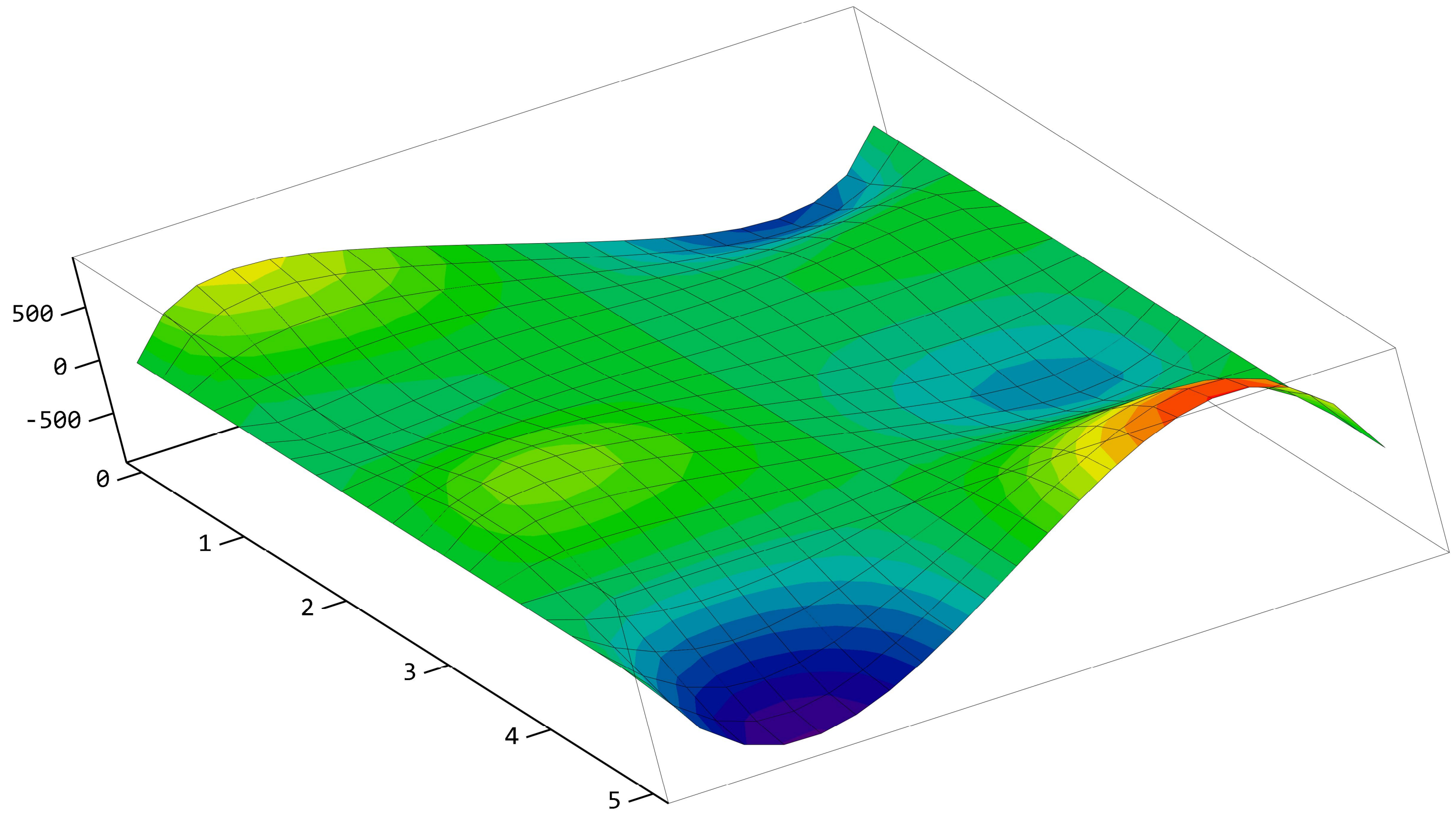
Funkcje momentów zginających:

$$Mx(x,y) := D0 \cdot \left[\sum_i \left[\left[(\alpha_i)^2 \cdot f0(i,y) - \nu \cdot f2(i,y)\right] \cdot \sin(\alpha_i \cdot x)\right]\right]$$

$$My(x,y) := D0 \cdot \left[\sum_i \left[\left[\nu \cdot (\alpha_i)^2 \cdot f0(i,y) - f2(i,y)\right] \cdot \sin(\alpha_i \cdot x)\right]\right]$$

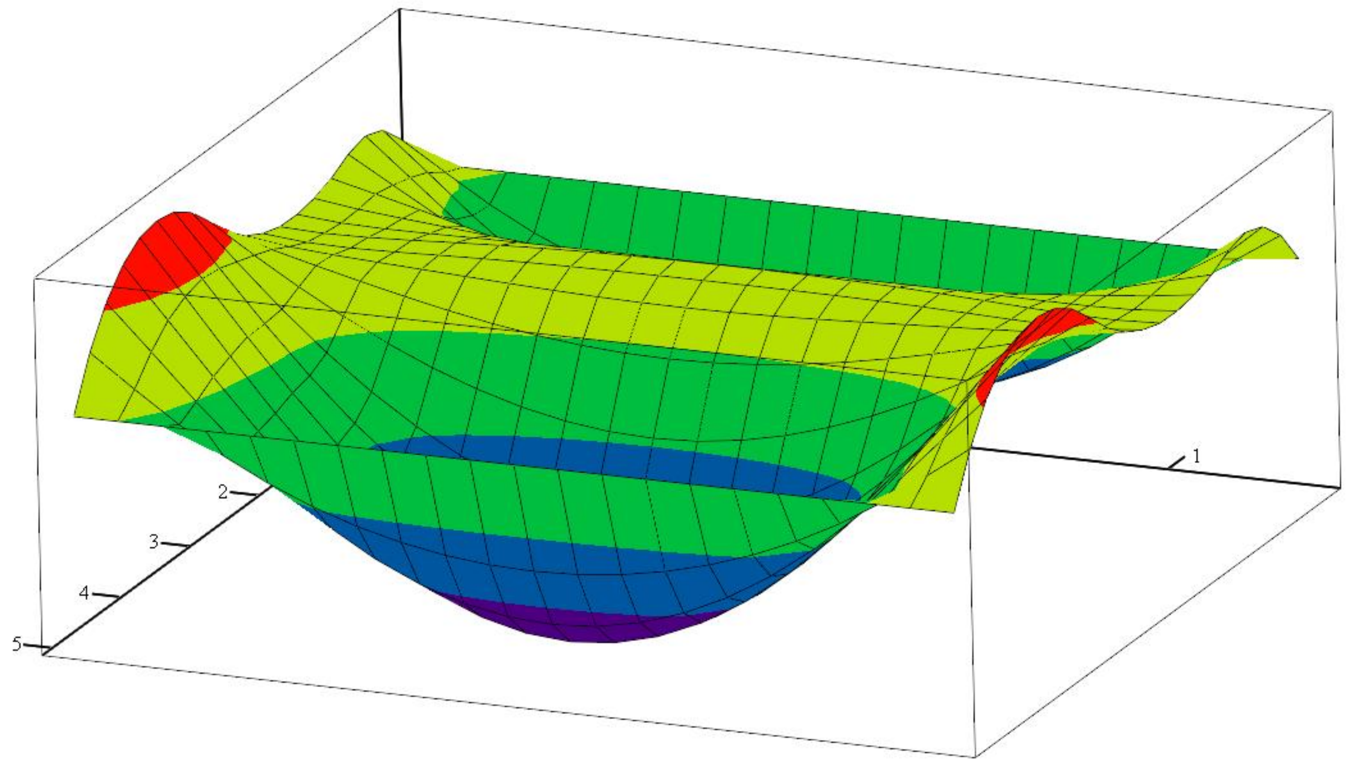
Funkcja momentu skręcającego:

$$Mxy(x,y) := -D0 \cdot (1-\nu) \cdot \left[\sum_i \left(\alpha_i \cdot f1(i,y) \cdot \cos(\alpha_i \cdot x)\right)\right]$$



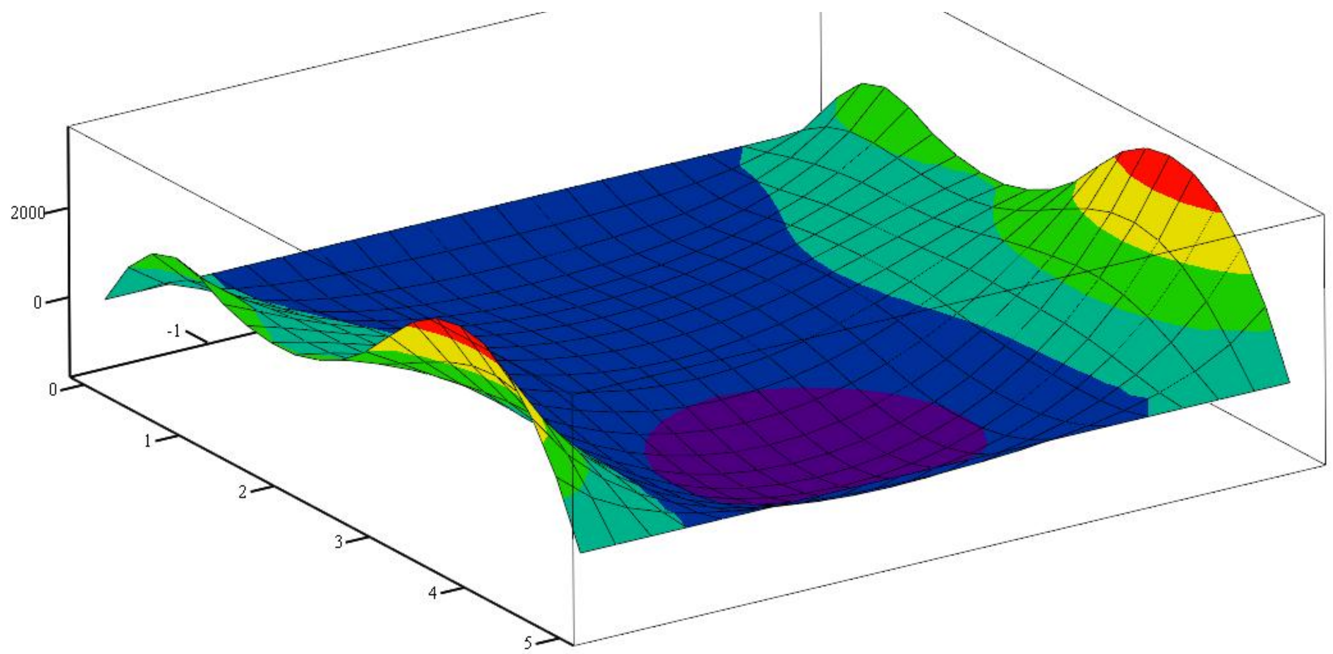
Mxy

$$Mxy\left(Lx, \frac{Ly}{4}\right) = 0.84509 \cdot \frac{kN \cdot m}{m}$$



$$Mx\left(\frac{Lx}{2},0\right) = 0.18178 \cdot \frac{kN \cdot m}{m}$$

Mx



$$My\left(\frac{Lx}{2},0\right) = -0.84144 \cdot \frac{kN \cdot m}{m}$$

My