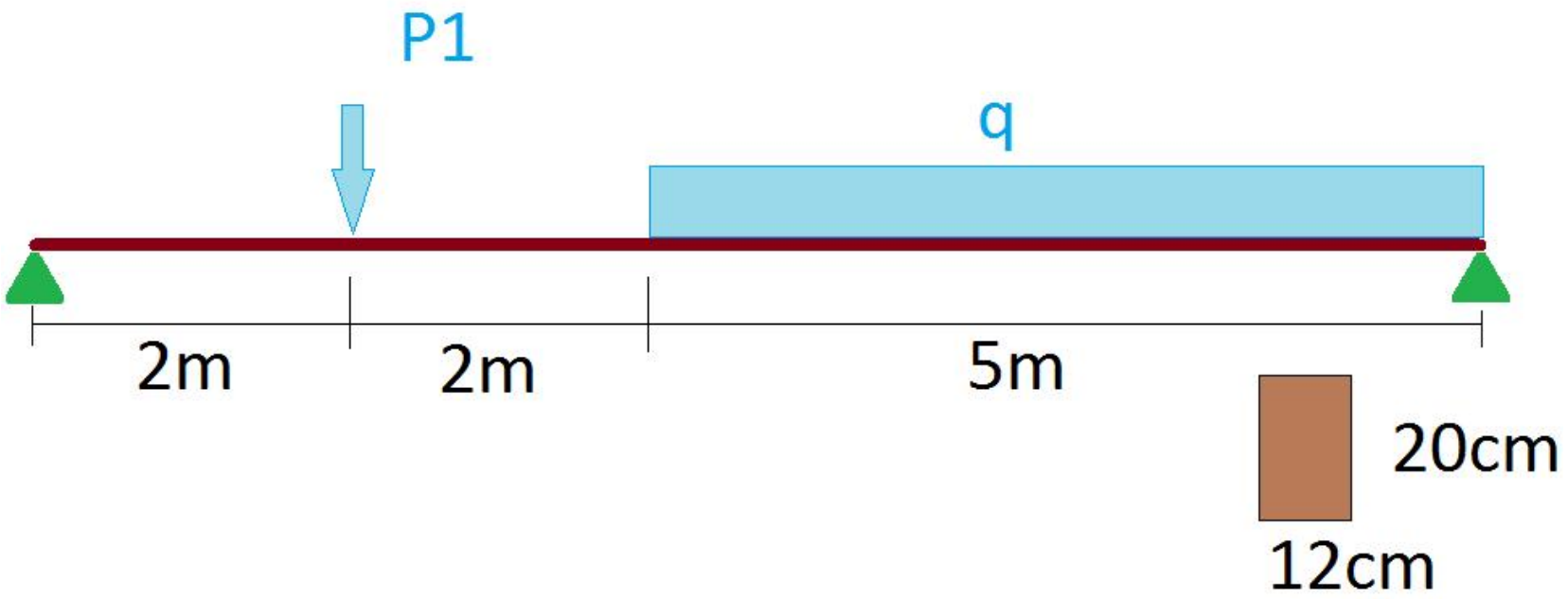
# *Beam deflection by Finite Difference Method*




ORIGIN := 0

E := 12GPa - *Young modulus for wood*

b := 12cm - *Cross section breadth*

h := 20cm - *Cross section height*

$J := b \cdot \frac{h^3}{12}$ $J = 8000.000 \cdot cm^4$ - *Moment of the inertia for the beam cross section*

$P1 := 7kN$

$q := 3\frac{kN}{m}$

$L1 := 2m \quad L2 := 5m \quad L := 9m$

$n := 9$

$\Delta := \frac{L}{n} \qquad \alpha := \frac{\Delta^2}{E \cdot J}$

*Reaction forces in the support*

$R0 := P1 \cdot \frac{(L1 + L2)}{L} + q \cdot \frac{L2^2}{2 \cdot L} \qquad R0 = 9.611 \cdot kN$

$R9 := P1 + q \cdot L2 - R0 \qquad R9 = 12.389 \cdot kN$

## *Bending moments functions*

$M1(x) := R0 \cdot x$ - $x \in \langle 0 \ldots L1 \rangle$

$M2(x) := M1(x) - P1 \cdot (x - L1)$ - $x \in \langle L1 \ldots 2L1 \rangle$

$M3(x) := M2(x) - q \cdot \frac{(x - 2 \cdot L1)^2}{2}$ - $x \in \langle 2L1 \ldots L \rangle$

$i := 0 \ldots n \quad X_i := i \cdot \Delta$ *nodal points co-ordinates*

## *Bending moments matrix*

$i := 0 \ldots 2 \quad M_i := M1(X_i)$

$i := 3 \ldots 4 \quad M_i := M2(X_i)$

$i := 5 \ldots n \quad M_i := M3(X_i)$

$M =$

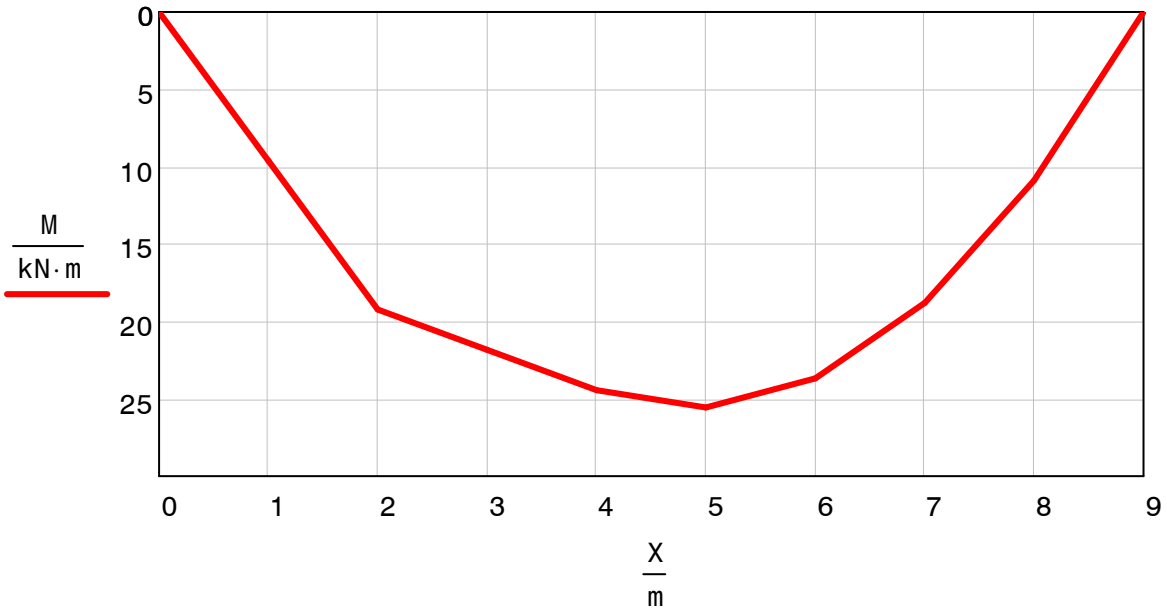
| | 0 |
|---|---|
| 0 | 0.000 |
| 1 | 9.611 |
| 2 | 19.222 |
| 3 | 21.833 |
| 4 | 24.444 |
| 5 | 25.556 |
| 6 | 23.667 |
| 7 | 18.778 |
| 8 | 10.889 |
| 9 | 0.000 |

$\cdot kN \cdot m$ $X =$

| | 0 |
|---|---|
| 0 | 0.000 |
| 1 | 1.000 |
| 2 | 2.000 |
| 3 | 3.000 |
| 4 | 4.000 |
| 5 | 5.000 |
| 6 | 6.000 |
| 7 | 7.000 |
| 8 | 8.000 |
| 9 | 9.000 |

$m$

## *Bending moment graph*



*Creation of the finite difference matrix*

$i := 0 .. \quad n \qquad A_{i,\,i} := -2$

$i := 1 .. \quad n \qquad A_{i,\,i-1} := 1$

$i := 0 .. \quad n - 1 \qquad A_{i,\,i+1} := 1$

$A =$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 |
| 9 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 |

*Boudary conditions for free supported beam: $y_0 = 0$, $y_n = 0$*

$A_{0,\,0} := 1 \qquad A_{0,\,1} := 0$ *<-------modification of the row No 0*

$A_{n,\,n} := 1 \qquad A_{n,\,n-1} := 0$ *<-------modification of the row No n*

$A =$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 | 0.000 |
| 7 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 | 0.000 |
| 8 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | -2.000 | 1.000 |
| 9 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 |

*Solution by implicit method*

$y := \text{lsolve}(A, \alpha \cdot M)$

*Beam deflection*

$y =$

| | 0 |
|---|---|
| 0 | 0.000 |
| 1 | -7.944 |
| 2 | -14.886 |
| 3 | -19.826 |
| 4 | -22.492 |
| 5 | -22.612 |
| 6 | -20.069 |
| 7 | -15.062 |
| 8 | -8.098 |
| 9 | 0.000 |

$\cdot cm$