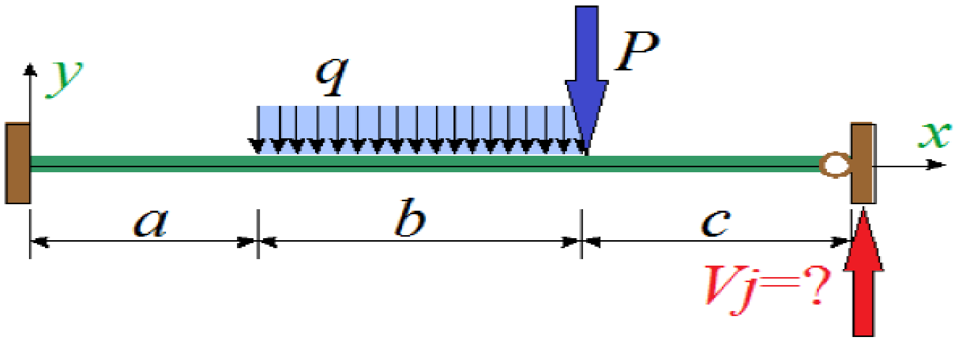
Należy obliczyć siłę węzłową Vj wykorzystując zasadę prac wirtualnych i funkcję kształtu elementu belkowego, dokładność $\pm 0.005 kN$

$q := 3 \cdot \frac{kN}{m}$ $\quad P := 4\ kN$ $\quad a := 2 \cdot m$ $\quad b := 4 \cdot m$ $\quad c := 1 \cdot m$ $\quad L := a + b + c$

$\xi 1 := \frac{a}{L} = 0.285714$ $\quad \xi 2 := \frac{a+b}{L} = 0.857143$ $\quad \xi P := \xi 2$



$$G0010(\xi) := \frac{\xi^2}{2} \cdot (3 - \xi)$$

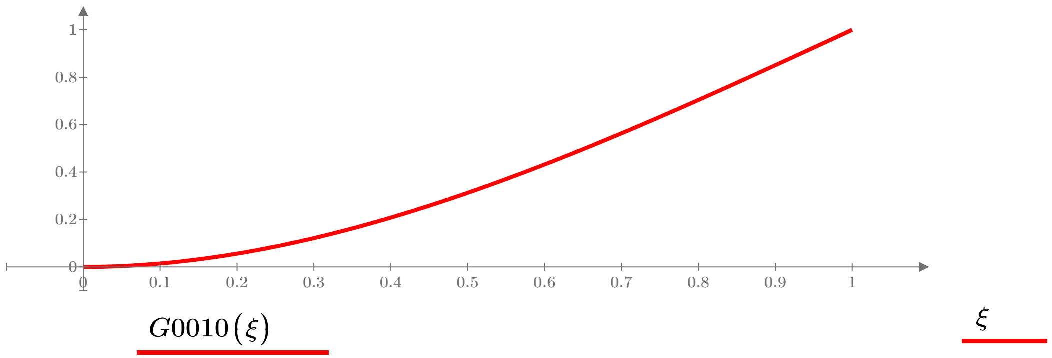
Równanie pracy wirtualnej

$$V_j \cdot \delta - P \cdot \delta \cdot G0010(\xi P) - \int_{x1}^{x2} q \cdot \delta \cdot G0010(\xi)\, \mathrm{d}x = 0$$

$$\Downarrow$$

$$\delta \cdot \left(V_j \cdot 1 - P \cdot G0010(\xi P) - q \cdot L \cdot \int_{\xi 1}^{\xi 2} G0010(\xi)\, \mathrm{d}\xi \right) = 0$$

$$V_j := q \cdot L \cdot \int_{\xi 1}^{\xi 2} G0010(\xi)\, \mathrm{d}\xi + P \cdot G0010(\xi P) = 8.12\ kN$$



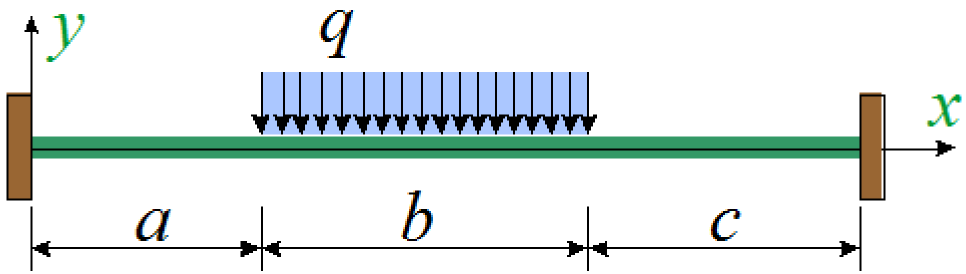
Definicja wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej

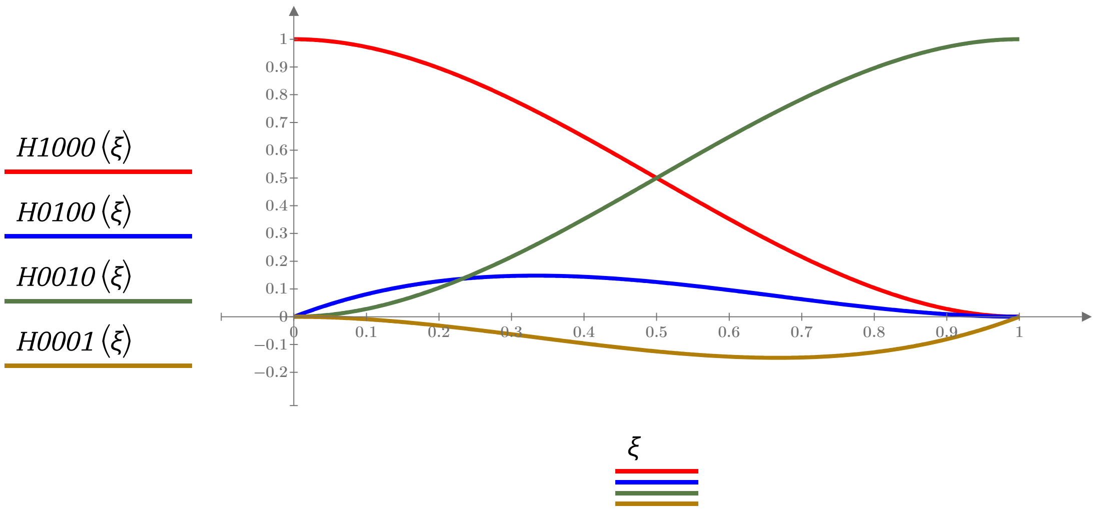
$$H1000(\xi) := 1 - 3\cdot\xi^2 + 2\cdot\xi^3$$

$$H0100(\xi) := \xi\cdot\left(1 - 2\cdot\xi + \xi^2\right)$$

$$H0010(\xi) := \xi^2\cdot(3 - 2\cdot\xi)$$

$$H0001(\xi) := -\xi^2\cdot(1 - \xi)$$





Wyprowadzenie wzoru $H1000(\xi)$

Warunki brzegowe:

$y(0) = \delta \qquad y'(0) = 0$

$y(l) = 0 \qquad y'(l) = 0$

$$y(x) = A + B\cdot x + C\cdot x^2 + D\cdot x^3$$

$$y'(x) = B + 2\ C\cdot x + 3\ D\cdot x^2$$

$y(0) = A = \delta \qquad y'(0) = B = 0$

$y(l) = \delta + C\cdot l^2 + D\cdot l^3 = 0$ ------> $C\cdot l + D\cdot l^2 = -\dfrac{\delta}{l}$

$y'(l) = 2\ C\cdot l + 3\ D\cdot l^2 = 0$ ------> $C\cdot l + \dfrac{3}{2}\ D\cdot l^2 = 0$

------> $\dfrac{1}{2}\ D\cdot l^2 = \dfrac{\delta}{l}$ ------> $D = \dfrac{2\ \delta}{l^3}$ ------> $C = \dfrac{-3\ \delta}{l^2}$

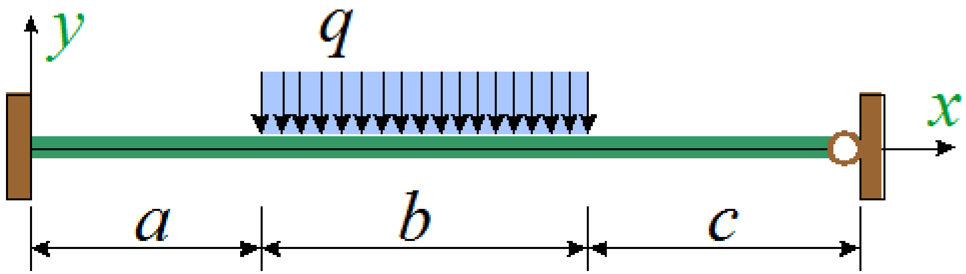
$y(x) = \delta\cdot\left(1 - 3\cdot\dfrac{x^2}{l^2} + 2\cdot\dfrac{x^3}{l^3}\right)$ ------> $y(\xi) = \delta\cdot\left(1 - 3\cdot\xi^2 + 2\cdot\xi^3\right)$ ------> $y(\xi) = \delta\cdot H1000(\xi)$ ------> $H1000(\xi) = \left(1 - 3\cdot\xi^2 + 2\cdot\xi^3\right)$

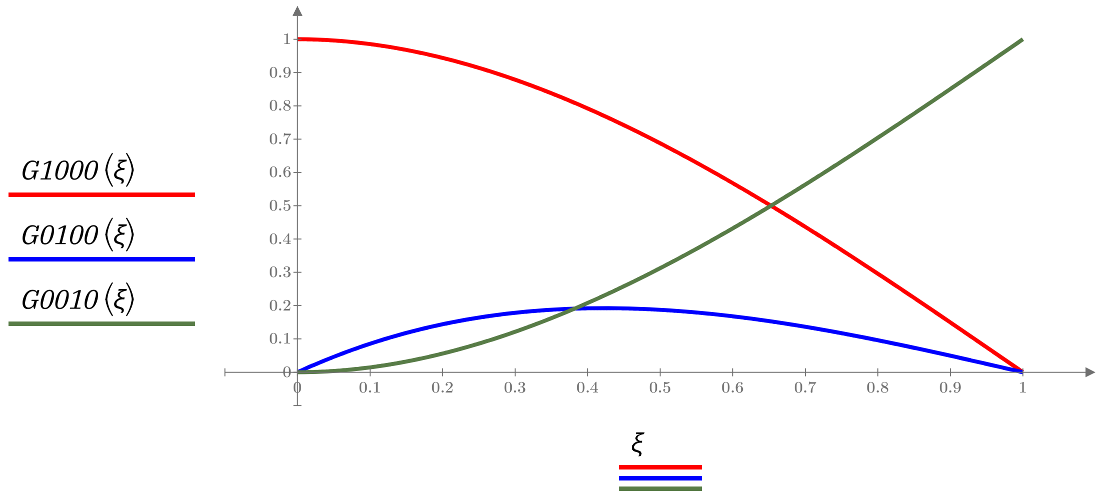
Definicja wielomianów Hermite'a dla belki połączonej przegubowo z prawym węzłem - G(ξ)

$$G1000(\xi) := 1 - \frac{3}{2} \cdot \xi^2 + \frac{1}{2} \cdot \xi^3$$

$$G0100(\xi) := \frac{\xi}{2} \cdot \left(2 - 3 \cdot \xi + \xi^2\right)$$

$$G0010(\xi) := \frac{\xi^2}{2} \cdot (3 - \xi)$$





Wyprowadzenie wzoru $G0100(\xi)$

Warunki brzegowe:

$y(0) = 0$ $\quad$ $y'(0) = \varphi$

$y(l) = 0$ $\quad$ $y''(l) = 0$

$$y(x) = A + B \cdot x + C \cdot x^2 + D \cdot x^3$$

$$y'(x) = B + 2\ C \cdot x + 3\ D \cdot x^2$$

$$y''(x) = 2\ C + 6\ D \cdot x$$

$y(0) = A = 0$ $\quad$ $y'(0) = B = \varphi$

$$\left.\begin{array}{l} y(l) = \varphi \cdot l + C \cdot l^2 + D \cdot l^3 = 0 \dashrightarrow C + D \cdot l = -\frac{\varphi}{l} \\ y''(l) = 2\ C + 6\ D \cdot l = 0 \dashrightarrow C + 3\ D \cdot l = 0 \end{array}\right\} \dashrightarrow 2\ D \cdot l = \frac{\varphi}{l^2} \dashrightarrow D = \frac{\varphi}{2\ l^2} \dashrightarrow C = \frac{-3\ \varphi}{2\ l}$$

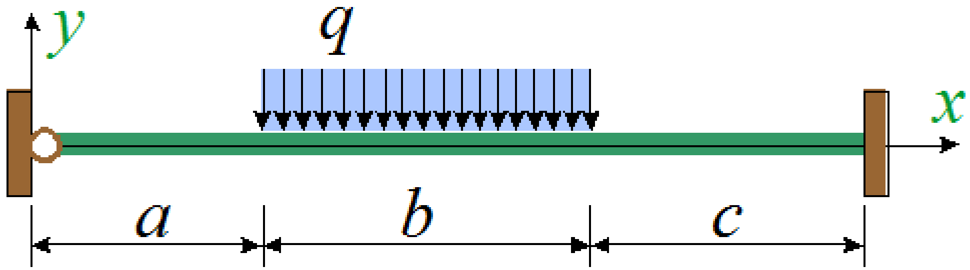
$$y(x) = \varphi \cdot \left(x - \frac{3}{2} \cdot \frac{x^2}{l} + \frac{x^3}{2\ l^2}\right) \dashrightarrow y(\xi) = \frac{\varphi \cdot l}{2} \left(2\ \xi - 3 \cdot \xi^2 + \xi^3\right) \dashrightarrow y(\xi) = \varphi \cdot l\ G0100(\xi) \dashrightarrow G0100(\xi) = \frac{\xi}{2} \cdot \left(2 - 3 \cdot \xi + \xi^2\right)$$

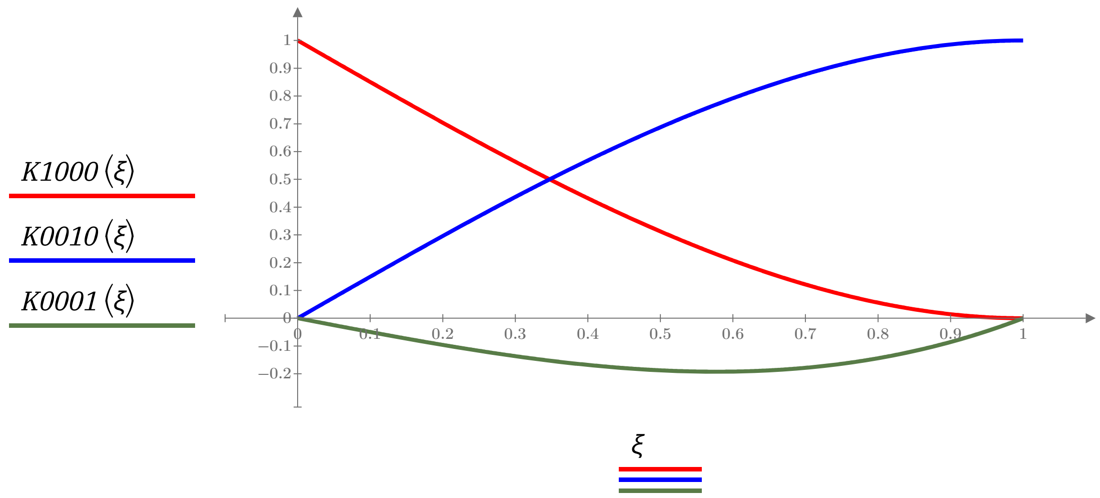
Definicja wielomianów Hermite'a dla belki połączonej przegubowo z lewym węzłem - K(ξ)

$$K1000(\xi):=1-\frac{3}{2}\cdot\xi+\frac{1}{2}\cdot\xi^3$$

$$K0010(\xi):=\frac{\xi}{2}\cdot\left(3-\xi^2\right)$$

$$K0001(\xi):=\frac{\xi}{2}\cdot\left(\xi^2-1\right)$$





Wyprowadzenie wzoru $K1000(\xi)$

Warunki brzegowe:

$y(0)=\delta$ $\quad$ $y''(0)=0$

$y(l)=0$ $\quad$ $y'(l)=0$

$$y(x)=A+B\cdot x+C\cdot x^2+D\cdot x^3$$

$$y'(x)=B+2\ C\cdot x+3\ D\cdot x^2$$

$$y''(x)=2\ C+6\ D\cdot x$$

$y(0)=A=\delta$ $\quad$ $y''(0)=2\ C=0$

$$y(l)=\delta+B\cdot l+D\cdot l^3=0 \text{ -----> } B+D\cdot l^2=-\frac{\delta}{l}$$

$$y'(l)=B+3\ D\cdot l^2=0 \text{ -----> } B+3\ D\cdot l^2=0$$

$$\text{-----> } 2\ D\cdot l^2=\frac{\delta}{l} \text{ -----> } D=\frac{\delta}{2\ l^3} \text{ -----> } B=\frac{-3\ \delta}{2\ l}$$

$$y(x)=\delta\cdot\left(1-\frac{3}{2}\cdot\frac{x}{l}+\frac{x^3}{2\ l^3}\right) \text{ -----> } y(\xi)=\delta\left(1-\frac{3}{2}\cdot\xi+\frac{1}{2}\ \xi^3\right) \text{ -----> } y(\xi)=\delta\ K1000(\xi) \text{ -----> } K1000(\xi)=1-\frac{3}{2}\cdot\xi+\frac{1}{2}\ \xi^3$$