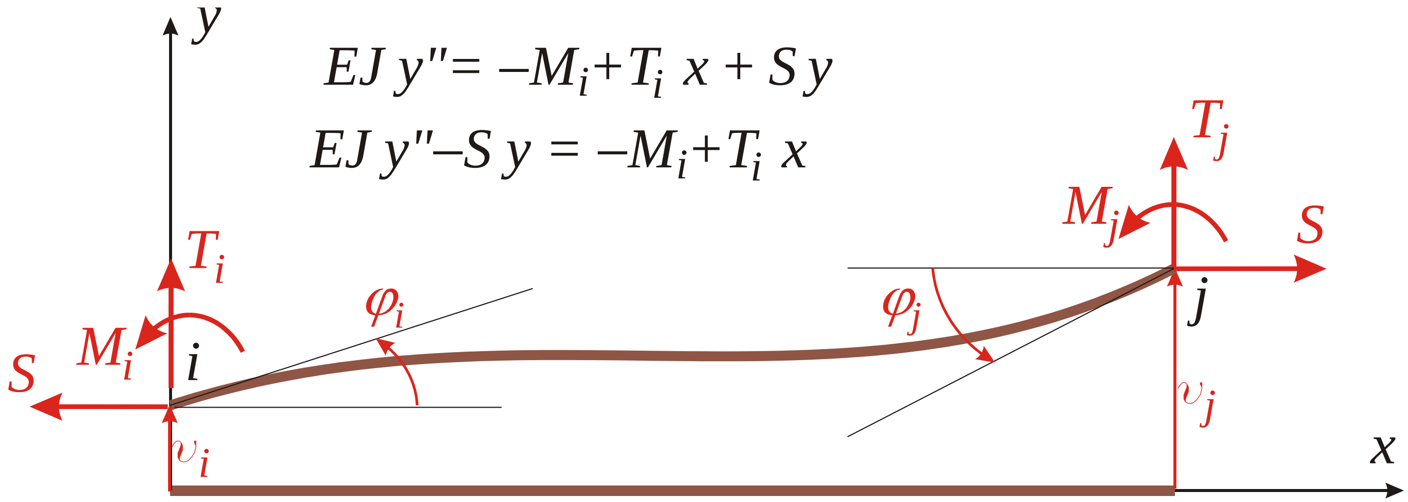
$$EJ\,y'' = -M_i + T_i\,x + S\,y$$

$$EJ\,y'' - S\,y = -M_i + T_i\,x$$



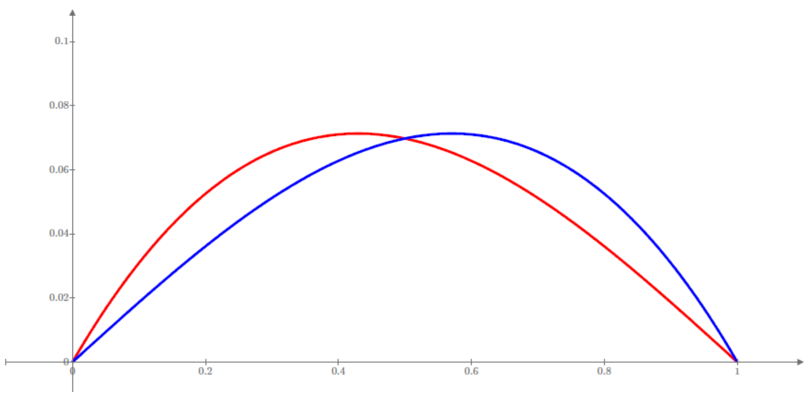
$$y(x) = A + B\xi + C\sinh(\alpha\xi) + D\cosh(\alpha\xi)\,, \qquad S>0$$

$$y(x) = A + B\xi + C\sin(\alpha\xi) + D\cos(\alpha\xi)\,, \qquad S<0 \qquad\qquad \alpha = l\sqrt{\frac{S}{EJ}} \qquad l\xi = x$$

$$\mu i(\alpha,\xi) := \frac{\sin(\alpha\cdot(1-\xi))}{\sin(\alpha)} - 1 + \xi \qquad \mu j(\alpha,\xi) := \frac{\sin(\alpha\cdot\xi)}{\sin(\alpha)} - \xi \qquad \mu i(\alpha,\xi) := \frac{-\sinh(\alpha\cdot(1-\xi))}{\sinh(\alpha)} + 1 - \xi \qquad \mu j(\alpha,\xi) := \frac{-\sinh(\alpha\cdot\xi)}{\sinh(\alpha)} + \xi$$

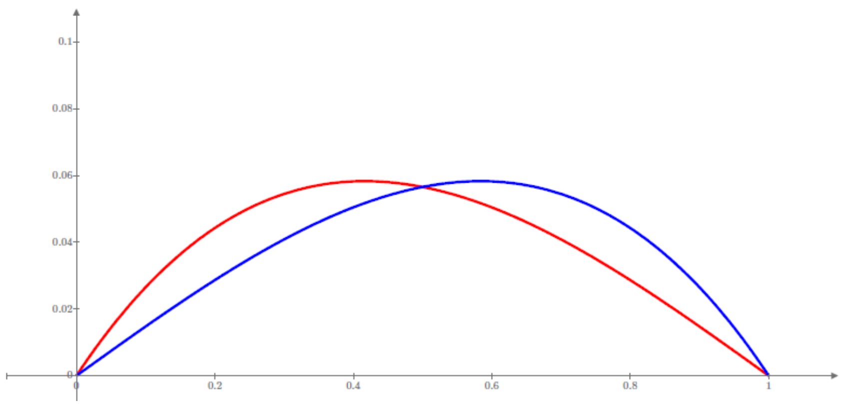
$\mu i(1,\xi)$

$\mu j(1,\xi)$



$\mu i(1,\xi)$

$\mu j(1,\xi)$



$$M_i = \frac{EJ}{l}\left[c(\alpha)\varphi_i + s(\alpha)\varphi_j - r(\alpha)\frac{u_j - u_i}{l}\right] \qquad T_i = -\frac{EJ}{l^2}\left[r(\alpha)\varphi_i + r(\alpha)\varphi_j - b(\alpha)\frac{u_j - u_i}{l}\right]$$

$$M_j = \frac{EJ}{l}\left[s(\alpha)\varphi_i + c(\alpha)\varphi_j - r(\alpha)\frac{u_j - u_i}{l}\right] \qquad T_j = \frac{EJ}{l^2}\left[r(\alpha)\varphi_i + r(\alpha)\varphi_j - b(\alpha)\frac{u_j - u_i}{l}\right]$$

$$c(\alpha) = \frac{\alpha}{d(\alpha)}(\sin\alpha - \alpha\cos\alpha) \qquad d(\alpha) = 2(1-\cos\alpha) - \alpha\sin\alpha$$

$$s(\alpha) = \frac{\alpha}{d(\alpha)}(\alpha - \sin\alpha) \qquad r(\alpha) = s(\alpha) + c(\alpha) = \frac{\alpha^2}{d(\alpha)}(1-\cos\alpha) \qquad b(\alpha) = \frac{\alpha^3}{d(\alpha)}\sin\alpha$$

$$M_i = \frac{EJ}{l}\left[c^I(\alpha)\varphi_i - c^I(\alpha)\frac{u_j - u_i}{l}\right] \qquad T_i = -\frac{EJ}{l^2}\left[c^I(\alpha)\varphi_i - r^I(\alpha)\frac{u_j - u_i}{l}\right]$$

$$M_j = 0 \qquad T_j = \frac{EJ}{l^2}\left[c^I(\alpha)\varphi_i - r^I(\alpha)\frac{u_j - u_i}{l}\right]$$

$$c^I(\alpha) = \alpha^2\frac{\sin\alpha}{\sin\alpha - \alpha\cos\alpha} \qquad r^I(\alpha) = \alpha^3\frac{\cos\alpha}{\sin\alpha - \alpha\cos\alpha}$$

$$\boldsymbol{K}(\sigma) = \frac{EJ}{l^2} \begin{array}{|c|c|c|c|} \hline \frac{\delta}{l} & \vartheta & -\frac{\delta}{l} & \vartheta \\ \hline \vartheta & \alpha l & -\vartheta & \beta l \\ \hline -\frac{\delta}{l} & -\vartheta & \frac{\delta}{l} & -\vartheta \\ \hline \vartheta & \beta l & -\vartheta & \alpha l \\ \hline \end{array},$$

$$\begin{aligned} \text{gdzie}: \ \alpha &= \alpha(\sigma) = \frac{\sigma}{\Delta(\sigma)}(\sin\sigma - \sigma\cos\sigma), \\ \beta &= \beta(\sigma) = \frac{\sigma}{\Delta(\sigma)}(\sigma - \sin\sigma), \\ \vartheta &= \vartheta(\sigma) = \frac{\sigma^2}{\Delta(\sigma)}(1 - \cos\sigma), \\ \delta &= \delta(\sigma) = \frac{\sigma^3}{\Delta(\sigma)}\sin\sigma, \\ \Delta(\sigma) &= 2(1 - \cos\sigma) - \sigma\sin\sigma. \end{aligned}$$

$$\sigma = l\sqrt{\frac{S}{EJ}}$$

$$\begin{aligned} \sin\sigma &= \sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \frac{\sigma^7}{7!} + \cdots, \\ \cos\sigma &= 1 - \frac{\sigma^2}{2!} + \frac{\sigma^4}{4!} - \frac{\sigma^6}{6!} + \cdots. \end{aligned}$$

$$\Delta(\sigma) = 2\left(1 - 1 + \frac{\sigma^2}{2!} - \frac{\sigma^4}{4!} + \frac{\sigma^6}{6!} - \cdots\right) - \sigma\left(\sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \frac{\sigma^7}{7!} + \cdots\right) =$$

$$= \sigma^4\left(\frac{1}{12} - \frac{\sigma^2}{180} + \frac{\sigma^4}{6720} - \frac{\sigma^6}{453600} + \cdots\right),$$

$$\alpha(\sigma) = \frac{\sigma}{\Delta(\sigma)}\left(\sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \frac{\sigma^7}{7!} + \cdots - \sigma + \frac{\sigma^3}{2!} - \frac{\sigma^5}{4!} + \frac{\sigma^7}{6!} - \cdots\right) =$$

$$= \frac{\frac{1}{3} - \frac{\sigma^2}{30} + \frac{\sigma^4}{840} - \frac{\sigma^6}{45360} + \cdots}{\frac{1}{12} - \frac{\sigma^2}{180} + \frac{\sigma^4}{6720} - \frac{\sigma^6}{453600} + \cdots}.$$

Po wykonaniu dzielenia uzyskujemy wyrażenie:

$$\alpha(\sigma) = 4 - \frac{2}{15}\sigma^2 - \frac{11}{6300}\sigma^4 - \frac{1}{27000}\sigma^6 + \cdots. \tag{6–134}$$

W analogiczny sposób obliczamy:

$$\beta(\sigma) = 2 + \frac{1}{30}\sigma^2 + \frac{13}{12600}\sigma^4 + \frac{11}{378000}\sigma^6 + \cdots,$$

$$\vartheta(\sigma) = 6 - \frac{1}{10}\sigma^2 - \frac{1}{1400}\sigma^4 - \frac{1}{126000}\sigma^6 - \cdots, \tag{6–135}$$

$$\delta(\sigma) = 12 - \frac{6}{5}\sigma^2 - \frac{1}{700}\sigma^4 - \frac{1}{63000}\sigma^6 - \cdots.$$

$$
\boldsymbol{K}(\sigma) = \frac{EJ}{l^2}\left\{ \begin{array}{|c|c|c|c|} \hline \frac{12}{l} & 6 & -\frac{12}{l} & 6 \\ \hline 6 & 4l & -6 & 2l \\ \hline -\frac{12}{l} & -6 & \frac{12}{l} & -6 \\ \hline 6 & 2l & -6 & 4l \\ \hline \end{array} - \frac{\sigma^2}{30} \begin{array}{|c|c|c|c|} \hline \frac{36}{l} & 3 & -\frac{36}{l} & 3 \\ \hline 3 & 4l & -3 & -l \\ \hline -\frac{36}{l} & -3 & \frac{36}{l} & -3 \\ \hline 3 & -l & -3 & 4l \\ \hline \end{array} - \right.
$$

$$
\left. - \frac{\sigma^4}{12600} \begin{array}{|c|c|c|c|} \hline \frac{18}{l} & 9 & -\frac{18}{l} & 9 \\ \hline 9 & 22l & -9 & 13l \\ \hline -\frac{18}{l} & -9 & \frac{18}{l} & -9 \\ \hline 9 & 13l & -9 & 22l \\ \hline \end{array} - \frac{\sigma^6}{378000} \begin{array}{|c|c|c|c|} \hline \frac{6}{l} & 3 & -\frac{6}{l} & 3 \\ \hline 3 & 14l & -3 & -11l \\ \hline -\frac{6}{l} & -3 & \frac{6}{l} & -3 \\ \hline 3 & -11l & -3 & 14l \\ \hline \end{array} \cdots \right\} \qquad (6\text{–}136)
$$