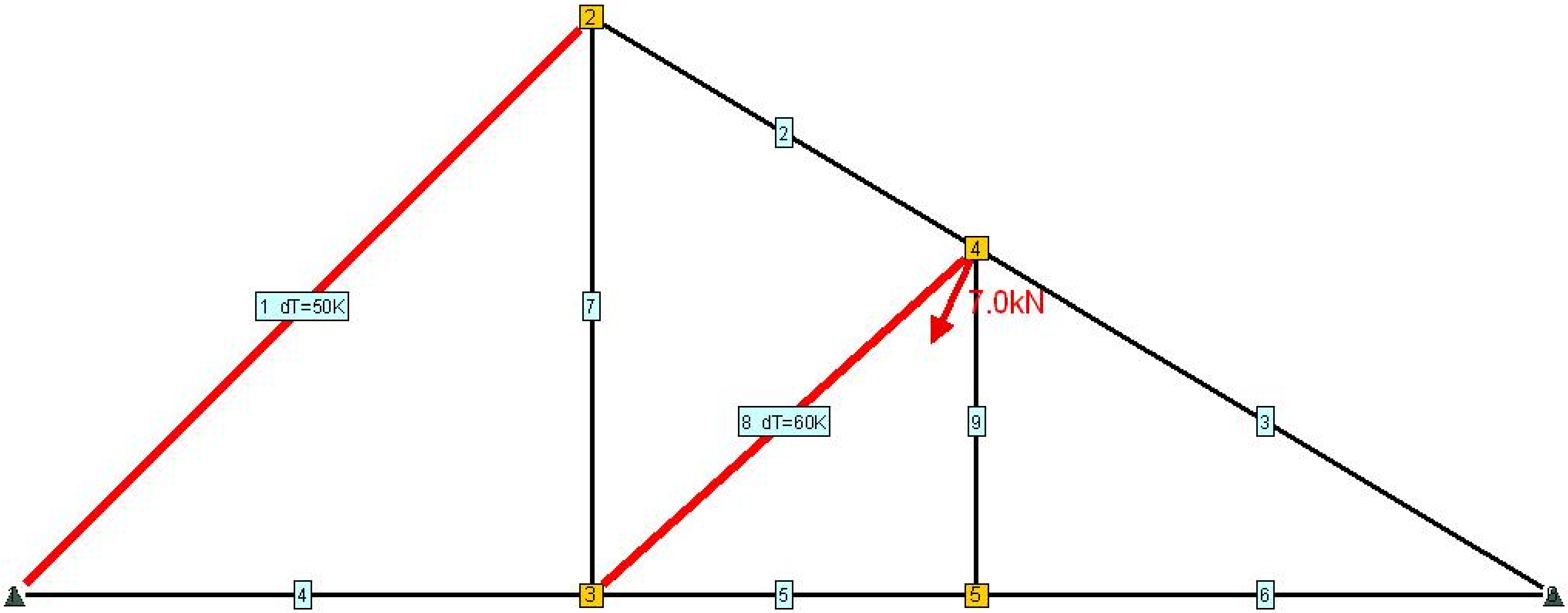
Statics of the truss with force and temperature load - test problem Nr 1



$E := 206GPa$ *- Young modulus for truss material - steel*

$\alpha_t := 10^{-5}$ *- thermal expanssion coeficient - steel*

$D := 5cm$ *- Cross section (pipe) diametr*

$g := 5mm$ *- Cross section (pipe) wall thicknes*

$A1 := \pi g \cdot (D - g)$ *- Cross sections area for elements*

$A1 = 7.069 \cdot cm^2$

Some main parameters of the truss

$Ne := 9$ *- Number of elements*

$Nn := 6$ *- Number of nodes*

$Nd := 2$ *- Degree of freedom for 2D truss node*

$Nq := Nd \cdot Nn$ *- Number of equilibrium equations*

$Ko_{Nq,Nq} := 0$ *- Initiation of the global stiffness matrix with zero values*

Element cross section area

$$A := \begin{pmatrix} A1 \\ A1 \\ A1 \\ A1 \\ A1 \\ A1 \\ A1 \\ A1 \\ A1 \end{pmatrix}$$

Global coordinates of the nodes

Element initial node (Ni) vector

Element final node (Nj) - vector

Temperature load for elements

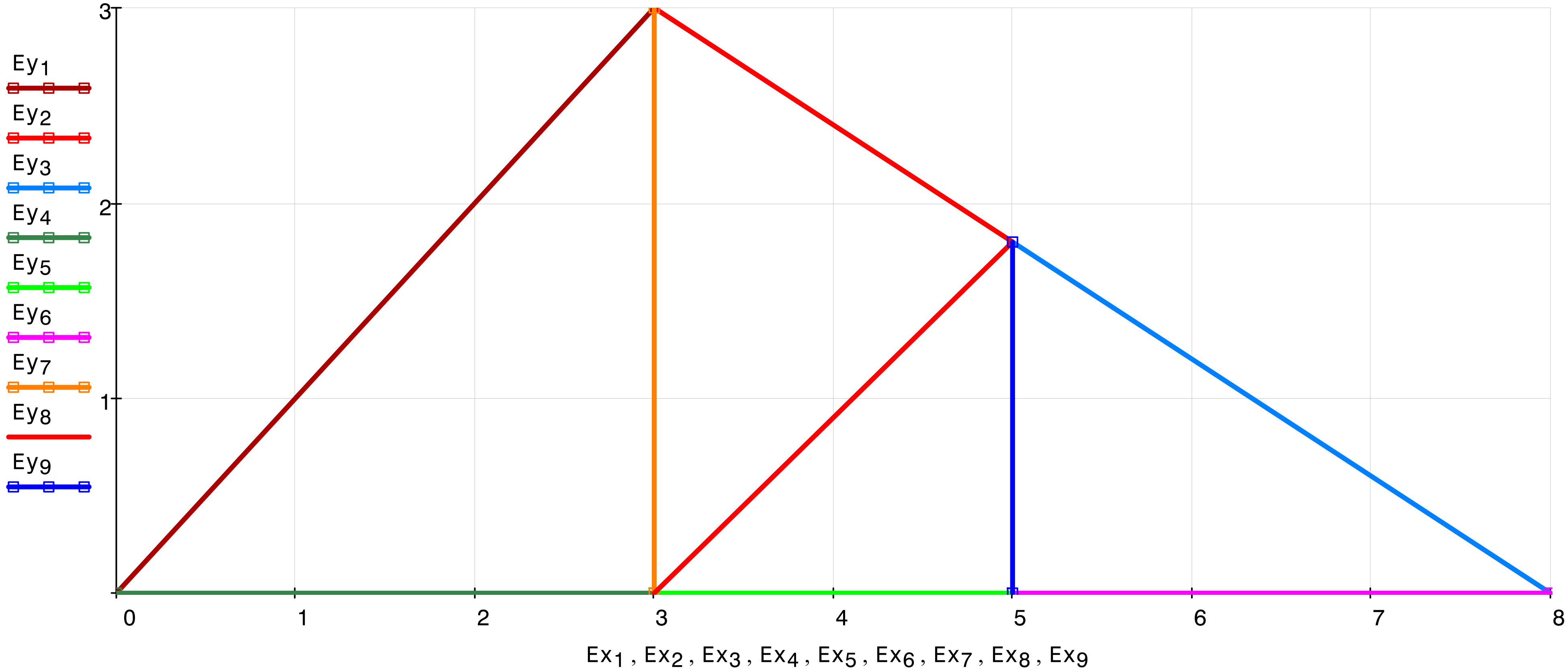
$$X := \begin{pmatrix} 0 \\ 3 \\ 3 \\ 5 \\ 5 \\ 8 \end{pmatrix} m \qquad Y := \begin{pmatrix} 0 \\ 3 \\ 0 \\ 1.8 \\ 0 \\ 0 \end{pmatrix} m \qquad Ni := \begin{pmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 3 \\ 5 \\ 3 \\ 3 \\ 5 \end{pmatrix} \qquad Nj := \begin{pmatrix} 2 \\ 4 \\ 6 \\ 3 \\ 5 \\ 6 \\ 2 \\ 4 \\ 4 \end{pmatrix} \qquad T := \begin{pmatrix} 50 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 60 \\ 0 \end{pmatrix}$$

$e := 1 .. \ Ne$ *Loop for each element*

Truss graph is made for easy geometrical data verification

$$Ex_e := \begin{bmatrix} X_{(Ni_e)} \\ X_{(Nj_e)} \end{bmatrix} \qquad Ey_e := \begin{bmatrix} Y_{(Ni_e)} \\ Y_{(Nj_e)} \end{bmatrix}$$

Ex, Ey - coordinates of the truss element nodes



Ey1
Ey2
Ey3
Ey4
Ey5
Ey6
Ey7
Ey8
Ey9
3
2
1
0
1
2
3
4
5
6
7
8
Ex1 , Ex2 , Ex3 , Ex4 , Ex5 , Ex6 , Ex7 , Ex8 , Ex9

*Calculation of the stiffness matrix blocks (**J**) for each element*

$$Lx_e := X_{(Nj_e)} - X_{(Ni_e)}$$

$$Ly_e := Y_{(Nj_e)} - Y_{(Ni_e)}$$

$$L_e := \sqrt{(Lx_e)^2 + (Ly_e)^2}$$

Lx = (m)

	1
1	3.000
2	2.000
3	3.000
4	3.000
5	2.000
6	3.000
7	0.000
8	2.000
9	0.000

Ly = (m)

	1
1	3.000
2	-1.200
3	-1.800
4	0.000
5	0.000
6	0.000
7	3.000
8	1.800
9	1.800

L = (m)

	1
1	4.243
2	2.332
3	3.499
4	3.000
5	2.000
6	3.000
7	3.000
8	2.691
9	1.800

$$J_e := \frac{E \cdot A_e}{(L_e)^3} \cdot \begin{bmatrix} (Lx_e)^2 & Lx_e \cdot Ly_e \\ Lx_e \cdot Ly_e & (Ly_e)^2 \end{bmatrix}$$

$$J_1 = \begin{pmatrix} 17160.6 & 17160.6 \\ 17160.6 & 17160.6 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_2 = \begin{pmatrix} 45905.1 & -27543.1 \\ -27543.1 & 16525.8 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_3 = \begin{pmatrix} 30603.4 & -18362.1 \\ -18362.1 & 11017.2 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_4 = \begin{pmatrix} 48537.6 & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_5 = \begin{pmatrix} 72806.4 & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_6 = \begin{pmatrix} 48537.6 & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_7 = \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 48537.6 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_8 = \begin{pmatrix} 29898.7 & 26908.8 \\ 26908.8 & 24217.9 \end{pmatrix} \cdot \frac{kN}{m}$$

$$J_9 = \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 80896.0 \end{pmatrix} \cdot \frac{kN}{m}$$

Function MBL - Matrix Block Location, this function is used for global stiffness matrix agregation

```
MBL(A, B, r, c) := | for  i ∈ 0 .. rows(B) − 1
                   |   for  j ∈ 0 .. cols(B) − 1
                   |     A[r+i, c+j] ← B[1+i, 1+j]        <----- r = row number, c =column number for B block location
                   | A
```

$$MBL(A,B,r,c) := \left| \begin{array}{l} \text{for } i \in 0..\ rows(B)-1 \\ \quad \text{for } j \in 0..\ cols(B)-1 \\ \quad\quad A_{r+i,\,c+j} \leftarrow B_{1+i,\,1+j} \\ A \end{array} \right.$$

<----- r = row number, c =column number for B block location

Agregation of the global stiffness matrix

$n_e := Nd \cdot Ni_e - 1$ $\qquad$ $k_e := Nd \cdot Nj_e - 1$

<--- n_e = global number of degree of freedom of the initial node,
<--- k_e = global number of degree of freedom of the final node.

In the agregation procedure, the user defined function - MBL is used

$$K := \sum_e \left(MBL\left(K_O, J_e, n_e, n_e\right) + MBL\left(K_O, J_e, k_e, k_e\right) - MBL\left(K_O, J_e, n_e, k_e\right) - MBL\left(K_O, J_e, k_e, n_e\right)\right)$$

K =

	1	2	3	4	5	6	7	8	9	10	11	12
1	65698.2	17160.6	-17160.6	-17160.6	-48537.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	17160.6	17160.6	-17160.6	-17160.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	-17160.6	-17160.6	63065.8	-10382.4	0.0	0.0	-45905.1	27543.1	0.0	0.0	0.0	0.0
4	-17160.6	-17160.6	-10382.4	82224.1	0.0	-48537.6	27543.1	-16525.8	0.0	0.0	0.0	0.0
5	-48537.6	0.0	0.0	0.0	151242.7	26908.8	-29898.7	-26908.8	-72806.4	0.0	0.0	0.0
6	0.0	0.0	0.0	-48537.6	26908.8	72755.5	-26908.8	-24217.9	0.0	0.0	0.0	0.0
7	0.0	0.0	-45905.1	27543.1	-29898.7	-26908.8	106407.2	-18996.3	0.0	0.0	-30603.4	18362.1
8	0.0	0.0	27543.1	-16525.8	-26908.8	-24217.9	-18996.3	132657.0	0.0	-80896.0	18362.1	-11017.2
9	0.0	0.0	0.0	0.0	-72806.4	0.0	0.0	0.0	121344.0	0.0	-48537.6	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-80896.0	0.0	80896.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	-30603.4	18362.1	-48537.6	0.0	79141.0	-18362.1
12	0.0	0.0	0.0	0.0	0.0	0.0	18362.1	-11017.2	0.0	0.0	-18362.1	11017.2

$\cdot \frac{kN}{m}$

Global stiffness matrix **K** *without boundary condition is singular* $|\mathbf{K}| = 0$

$$\left| K \cdot \frac{1m}{kN} \right| = 0.000$$

<----- by the small errors in computer arithmetic,
the value of the determinant can be different a bit from a zero

Global vektor of external forces - right hand side (RHS) vector

Horizontal and vertical projection of the force acting in node 8 (7kN)

$$Fx_4 := -7kN \cdot \sin(25deg) = -2.958 \cdot kN$$

$$Fy_4 := -7kN \cdot \cos(25deg) = -6.344 \cdot kN$$

$$p := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Fx_4 \\ Fy_4 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

p = (in kN)

	1
1	0.000
2	0.000
3	0.000
4	0.000
5	0.000
6	0.000
7	-2.958
8	-6.344
9	0.000
10	0.000
11	0.000
12	0.000

· kN

- Nodal forces from temperature load in element "e"

$$t_e := \alpha_t \cdot T_e \cdot \frac{E \cdot A_e}{L_e} \begin{pmatrix} Lx_e \\ Ly_e \end{pmatrix} \qquad pT_{0_{Nq}} := 0$$

Agregation of the thermal force vector **p**T

$$pT := \sum_e \left(MBL\left(pT_0, t_e, n_e, 1\right) - MBL\left(pT_0, t_e, k_e, 1\right)\right)$$

$pT^T =$

	1	2	3	4	5	6	7	8	9	10	11	12
1	51.482	51.482	-51.482	-51.482	64.940	58.446	-64.940	-58.446	0.000	0.000	0.000	0.000

$\cdot kN$

*Copy of the **K** matrix and **p** wector*

$K_o := K$ $\qquad$ $p_o := p - pT$

Boundary conditions

node No 1: degree of freedom s1 i s2 $\qquad$ *node No 6: degree of freedom s3 i s4*

$s1 := 1$ $\qquad$ $s2 := 2$ $\qquad$ $s3 := 11$ $\qquad$ $s4 := 12$

$i := 1 .. Nq$

$K_{o_{s1,i}} := 0$ $\qquad$ $K_{o_{s2,i}} := 0$ $\qquad$ $K_{o_{s3,i}} := 0$ $\qquad$ $K_{o_{s4,i}} := 0$ $\qquad$ *putting zero values in the K matrix rows*

$K_{o_{i,s1}} := 0$ $\qquad$ $K_{o_{i,s2}} := 0$ $\qquad$ $K_{o_{i,s3}} := 0$ $\qquad$ $K_{o_{i,s4}} := 0$ $\qquad$ *putting zero values in the K matrix columns*

$K_{o_{s1,s1}} := 1\frac{kN}{m}$ $\qquad$ $K_{o_{s2,s2}} := 1\frac{kN}{m}$ $\qquad$ $K_{o_{s3,s3}} := 1\frac{kN}{m}$ $\qquad$ $K_{o_{s4,s4}} := 1\frac{kN}{m}$ $\qquad$ *putting 1 on the diagonal of stifness matrix*

$p_{o_{s1}} := 0$ $\qquad$ $p_{o_{s2}} := 0$ $\qquad$ $p_{o_{s3}} := 0$ $\qquad$ $p_{o_{s4}} := 0$ $\qquad$ *zero value for some rows in RHS vector*

$K_o =$

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	63065.8	-10382.4	0.0	0.0	-45905.1	27543.1	0.0	0.0	0.0	0.0
4	0.0	0.0	-10382.4	82224.1	0.0	-48537.6	27543.1	-16525.8	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	151242.7	26908.8	-29898.7	-26908.8	-72806.4	0.0	0.0	0.0
6	0.0	0.0	0.0	-48537.6	26908.8	72755.5	-26908.8	-24217.9	0.0	0.0	0.0	0.0
7	0.0	0.0	-45905.1	27543.1	-29898.7	-26908.8	106407.2	-18996.3	0.0	0.0	0.0	0.0
8	0.0	0.0	27543.1	-16525.8	-26908.8	-24217.9	-18996.3	132657.0	0.0	-80896.0	0.0	0.0
9	0.0	0.0	0.0	0.0	-72806.4	0.0	0.0	0.0	121344.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-80896.0	0.0	80896.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0

$\cdot \frac{kN}{m} p$

$\left| K_o \cdot 1 \frac{m}{kN} \right| = 1.536 \times 10^{38}$ *- determinant of the modified stiffness matrix* $\mathbf{K}_o$ *is always greather than 0,* $|\mathbf{K}_o| > 0$

Solving the system of linear equation: $u := \mathrm{lsolve}(K_o, p_o)$

u - vector of nodal displacements

$u^T =$

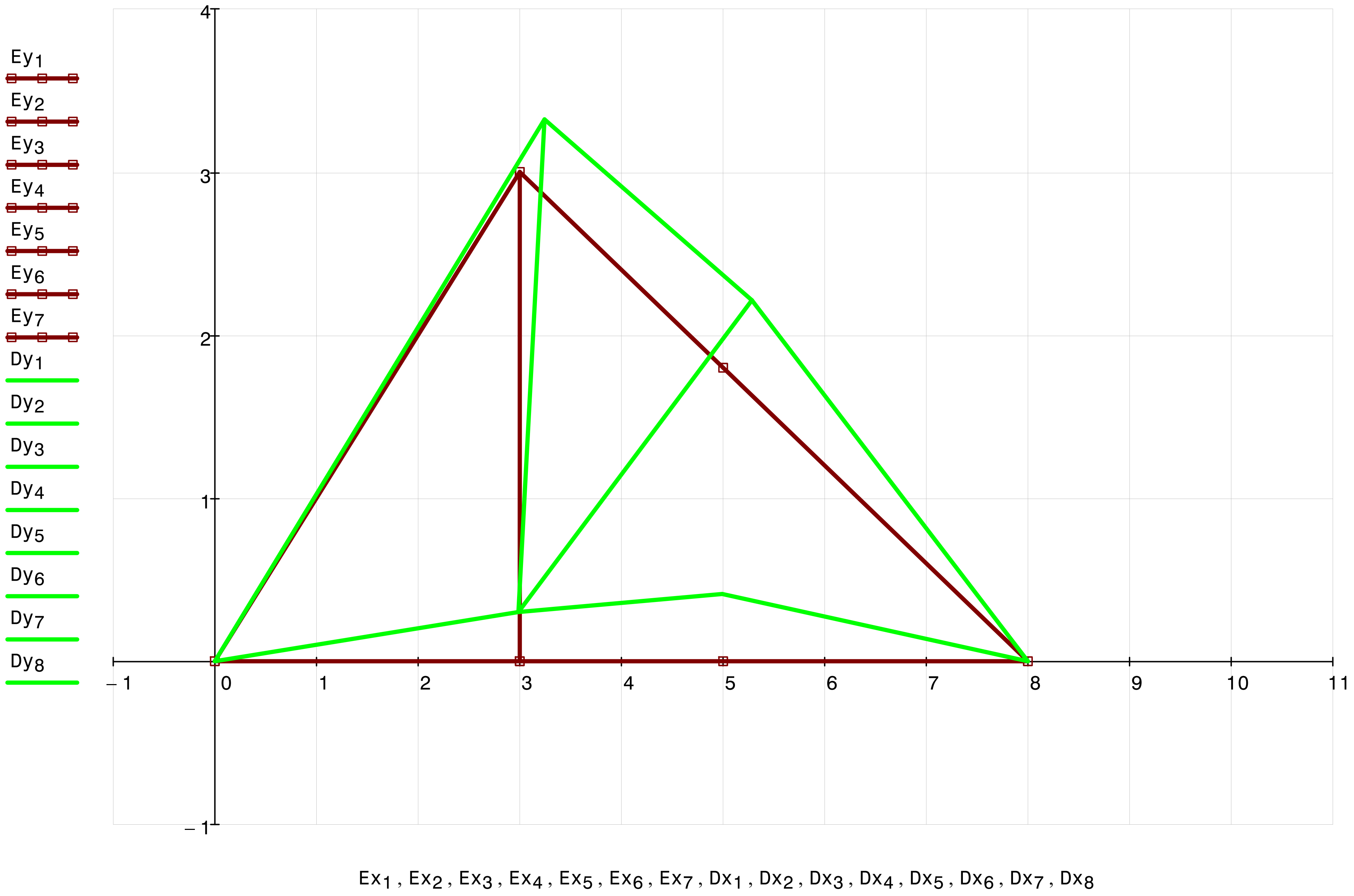
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0000	0.0000	1.2122	1.6104	-0.0697	1.5100	1.4199	2.0670	-0.0418	2.0670	0.0000	0.0000

$\cdot mm$

The graph of the displaced truss allows to check the correctness of results

$scale := 200$

$$Dx_e := Ex_e + scale \cdot \begin{bmatrix} u_{(2 \cdot Ni_e - 1)} \\ u_{(2 \cdot Nj_e - 1)} \end{bmatrix} \qquad Dy_e := Ey_e + scale \cdot \begin{bmatrix} u_{(2 \cdot Ni_e)} \\ u_{(2 \cdot Nj_e)} \end{bmatrix}$$



Ey1
Ey2
Ey3
Ey4
Ey5
Ey6
Ey7
Dy1
Dy2
Dy3
Dy4
Dy5
Dy6
Dy7
Dy8
4
3
2
1
0
−1
1
2
3
4
5
6
7
8
9
10
11
Ex1 , Ex2 , Ex3 , Ex4 , Ex5 , Ex6 , Ex7 , Dx1 , Dx2 , Dx3 , Dx4 , Dx5 , Dx6 , Dx7 , Dx8

Reaction of the supported nodes

$$r := K \cdot u - p + pT$$

$r^T =$

	1	2	3	4	5	6	7	8	9	10	11	12
1	6.428	3.045	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000	-0.000	-3.469	3.299

$\cdot kN$

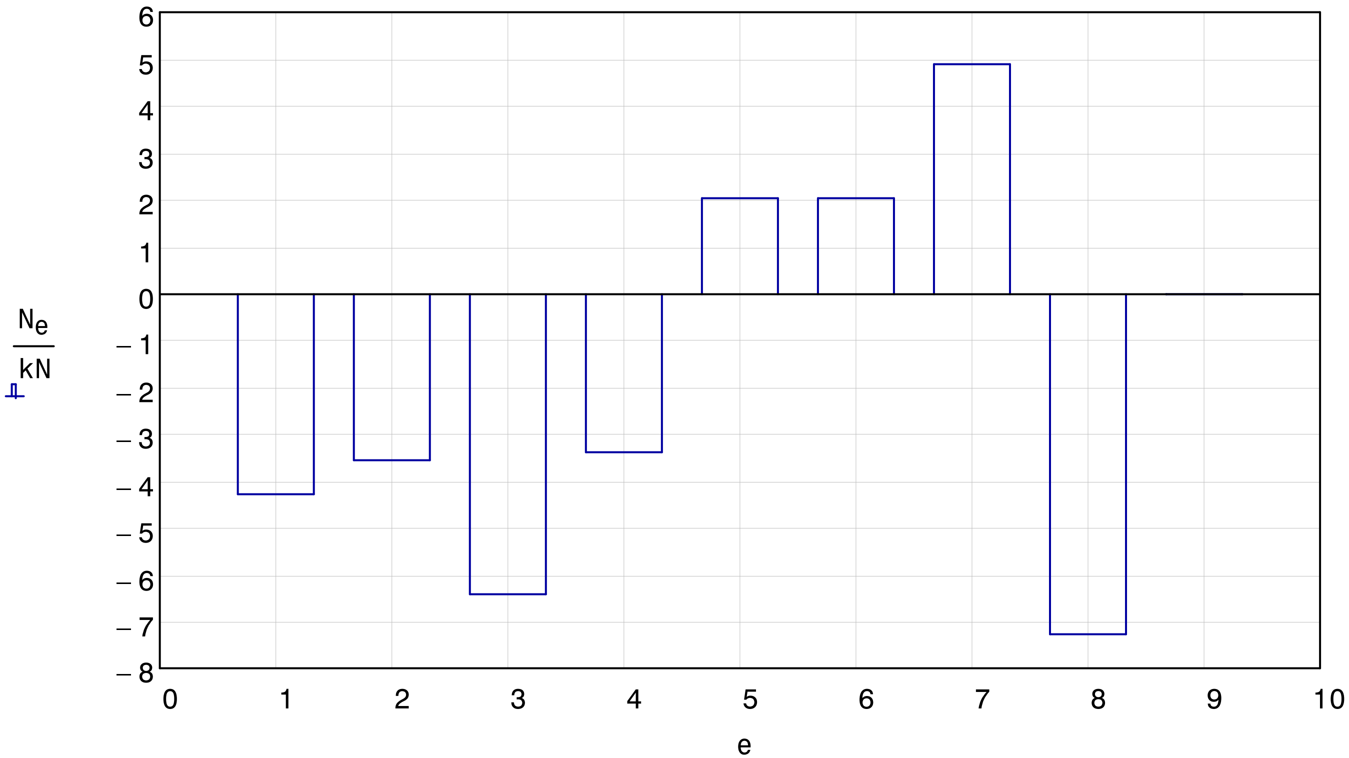
Calculation of the element internal forces

$$N_e := \frac{E \cdot A_e}{(L_e)^2} \cdot \left[\left(u_{2 \cdot Nj_e - 1} - u_{2 \cdot Ni_e - 1} \right) \cdot Lx_e + \left(u_{2 \cdot Nj_e} - u_{2 \cdot Ni_e} \right) \cdot Ly_e \right] - \alpha_t \cdot T_e \cdot E \cdot A_e$$

$N =$

	1
1	-4.306
2	-3.551
3	-6.413
4	-3.383
5	2.030
6	2.030
7	4.871
8	-7.282
9	0.000

$\cdot kN$



Calculation of the normal stress in the truss elements

$$\sigma_e := \frac{N_e}{A_e}$$

$\sigma =$

	1
1	-6.092
2	-5.023
3	-9.073
4	-4.786
5	2.872
6	2.872
7	6.892
8	-10.302
9	0.000

$\cdot \mathrm{MPa}$