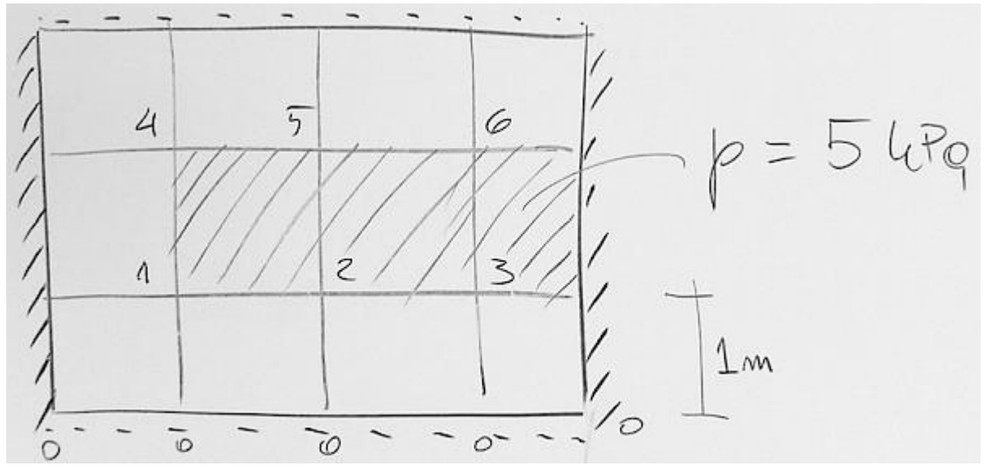
Obliczanie ugięcia płyty za pomocą metody różnic skończonych

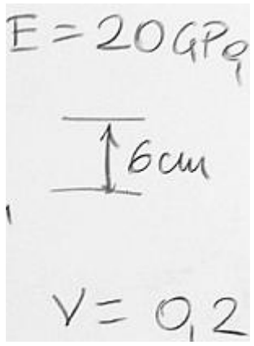
$ORIGIN := 0$

$E := 20GPa$ $\nu := 0.2$ $h := 6cm$ $Lx := 4m$ $Ly := 3m$ $\Delta := 1m$ $Nx := \frac{Lx}{\Delta} = 4$ $Ny := \frac{Ly}{\Delta} = 3$

$p0 := -4kPa$ - *obciążenie użytkowe*

$D := \frac{E \cdot h^3}{12\left(1 - \nu^2\right)} = 375 \cdot kN \cdot m$ - *sztywność płytowa*





$$N := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Tablica z numerami węzłów

$n := max(N) = 6$ $w_n := 0$

Tworzenie układu równań MRS

$A_{n,n} := 0$ $b_n := 0$

$i := 1 .. 2$ $j := 1 .. 3$

$$B := \sum_i \left(\sum_j Bilaplasjan(A, N, i, j) \right)$$

$i := 1 .. 6$ $b_i := \frac{1}{2}$

$b_1 := \frac{1}{4}$ $b_4 := \frac{1}{4}$

Warunek brzegowy w0=0

$i := 0$

$k := 0 .. n$ $B_{i,k} := 0$ $B_{i,i} := 1$ $b_i := 0$

Warunki brzegowe My=0 na brzegu y=0

$j := 1 .. 3$ $B_{j,j} := B_{j,j} - 1$

Warunki brzegowe My=0 na brzegu y=Ly

$j := 4 .. 6$ $B_{j,j} := B_{j,j} - 1$

Warunki brzegowe φy=0 na brzegu x=0

$j := 1, 4 .. 4$ $B_{j,j} := B_{j,j} + 1$

Warunki brzegowe φy=0 na brzegu x=Lx

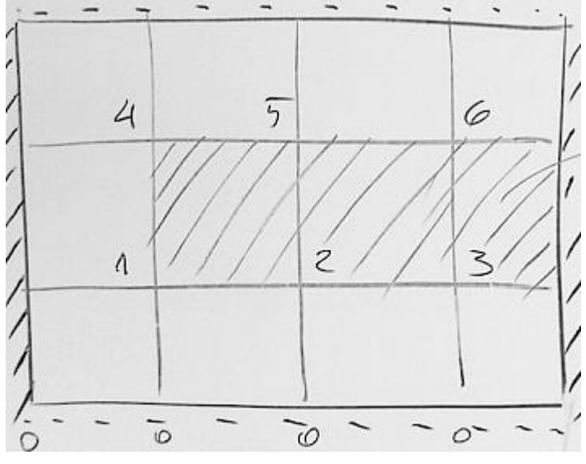
$j := 3, 6 .. 6$ $B_{j,j} := B_{j,j} + 1$

Wartości węzłowe prawej strony układu równań MRS

$$\alpha 0 := \frac{\Delta^4 \cdot p0}{D} = -10.666667 \cdot mm$$

$N =$

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	0
2	0	4	5	6	0
3	0	0	0	0	0



Układ równań MRS $B \cdot w = \alpha 0 \cdot b$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -9 & 20 & -8 & 1 & -8 & 2 & 0 \\ -1 & -8 & 19 & -8 & 2 & -8 & 2 \\ -9 & 1 & -8 & 20 & 0 & 2 & -8 \\ -9 & -8 & 2 & 0 & 20 & -8 & 1 \\ -1 & 2 & -8 & 2 & -8 & 19 & -8 \\ -9 & 0 & 2 & -8 & 1 & -8 & 20 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 0.25 \\ 0.5 \\ 0.5 \\ 0.25 \\ 0.5 \\ 0.5 \end{pmatrix}$$

$$\alpha 0 = -10.667 \cdot mm$$

$$w := lsolve(B, b \cdot \alpha 0) \qquad w = \begin{pmatrix} 0 \\ -0.94921 \\ -1.652582 \\ -1.191635 \\ -0.94921 \\ -1.652582 \\ -1.191635 \end{pmatrix} mm$$

$$
\begin{aligned}
&Bilaplasjan(A, N, i, j) := \\
&\quad a \leftarrow N_{i,j} \\
&\quad A_{a,a} \leftarrow A_{a,a} + 20 \\
&\quad A_{a,N_{i,j-1}} \leftarrow A_{a,N_{i,j-1}} - 8 \\
&\quad A_{a,N_{i,j+1}} \leftarrow A_{a,N_{i,j+1}} - 8 \\
&\quad A_{a,N_{i-1,j}} \leftarrow A_{a,N_{i-1,j}} - 8 \\
&\quad A_{a,N_{i+1,j}} \leftarrow A_{a,N_{i+1,j}} - 8 \\
&\quad A_{a,N_{i-1,j-1}} \leftarrow A_{a,N_{i-1,j-1}} + 2 \\
&\quad A_{a,N_{i+1,j-1}} \leftarrow A_{a,N_{i+1,j-1}} + 2 \\
&\quad A_{a,N_{i-1,j+1}} \leftarrow A_{a,N_{i-1,j+1}} + 2 \\
&\quad A_{a,N_{i+1,j+1}} \leftarrow A_{a,N_{i+1,j+1}} + 2 \\
&\quad A_{a,N_{i+2,j}} \leftarrow A_{a,N_{i+2,j}} + 1 \quad \text{if } i < Ny - 1 \\
&\quad A_{a,N_{i-2,j}} \leftarrow A_{a,N_{i-2,j}} + 1 \quad \text{if } i > 1 \\
&\quad A_{a,N_{i,j-2}} \leftarrow A_{a,N_{i,j-2}} + 1 \quad \text{if } j > 1 \\
&\quad A_{a,N_{i,j+2}} \leftarrow A_{a,N_{i,j+2}} + 1 \quad \text{if } j < Nx - 1 \\
&\quad A
\end{aligned}
$$